

AN INVESTIGATION OF TRANSISTORS  
AS VOLTAGE MULTIPLIERS

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ROBERT G. IVERSON  
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AS VOLTAGE AMPLIFIERS

by

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Lieutenant, U.S. Navy  
U.S. The United States Naval Academy (1945)

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U.S. The United States Naval Academy (1947)

Submitted to the Department of Naval Architecture  
and Marine Engineering on May 16, 1952 in partial  
fulfillment of the requirements for the degree of  
Naval Engineer.





## ABSTRACT

An Investigation of Transistors as Voltage Multipliers

by

Robert W. Iverson

and

Seymour F. Ross

Submitted to the Department of Naval Architecture and Marine Engineering on May 16, 1952 in partial fulfillment of the requirements for the degree of Naval Engineer.

The object of this thesis is to find out how well the function of voltage multiplication can be accomplished through the use of presently available transistors.

The problem is analyzed by expressing, in a Taylor series, the collector current as a function of the inputs (collector voltage and emitter current). Through analysis of the resulting series for sinusoidal inputs, the component of collector current at the sum or difference of the input frequencies is separated as

$$i_c = (a_0 + a_{12}\omega_c^2 + a_{11}\omega_c^2) e^{i\omega_c t}$$

These coefficients were experimentally determined for a standard Western Electric, Type 4-159B, transistor and a General Electric, Type 11, transistor on which the point-contact pressure had been lightened and to which

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Cambridge, Massachusetts  
May 15, 1952

Professor Leicester F. Hamilton  
Assistant Secretary of the Faculty  
Massachusetts Institute of Technology  
Cambridge, Massachusetts

Dear Sir:

In accordance with the requirements for the degree of Naval Engineer, we submit herewith a thesis entitled "An Investigation of Transistors as Voltage Multipliers".

Respectfully,

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### ACKNOWLEDGMENTS

The authors wish to acknowledge their indebtedness to Professor J. M. McIntyre for his advice and encouragement throughout the accomplishment of this investigation. They also wish to express their appreciation to Professor R. B. Adler for his constructive criticisms. The authors acknowledge at this time that, to the best of their knowledge, it was Mr. Earl Hurler, attached to the M.I.T. Electronics Research Laboratory, who first found that reducing the point-contact pressure of a transistor could measurably straighten the constant  $i_E$  curves shown on the collector characteristics.

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FROM : The Vice President

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3. The third part of the report is a presentation of the results of the study. It includes a discussion of the findings, a comparison of the results with previous research, and a discussion of the implications of the findings.

4. The fourth part of the report is a conclusion and a discussion of the limitations of the study. It includes a summary of the main findings and a discussion of the strengths and weaknesses of the study.

5. The fifth part of the report is a list of references. It includes a list of the books, articles, and other sources used in the study.

6. The sixth part of the report is an appendix. It includes a list of the tables, figures, and other supplementary material used in the study.

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9. The ninth part of the report is a list of footnotes. It includes a list of the footnotes used in the study.

10. The tenth part of the report is a list of appendices. It includes a list of the appendices used in the study.

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2. 3

## 1. INTRODUCTION

Because multiplication of two electrical signals, either voltages, currents, or a combination of the two, is an important operation in most electrical computation, many schemes have been devised to yield an output proportional to the product of the inputs. It is the purpose of this thesis to investigate the suitability of transistors for this operation.

The methods of multiplication can be roughly divided into frequency bands over which they are applicable. For instance, in the low frequency range, up to about 100 cps, many purely mechanical methods are available. One of the most accurate and best known methods is that employed by Bush and Salasoll<sup>(1)</sup>, which utilized the wheel and disc integrator. The product of  $x$  and  $y$  was formed in accordance with the equation:  $xy = \int x \, dy + \int y \, dx$ . The error of this method probably would not exceed one part in 25,000, but the equipment is very expensive.

A number of other mechanical methods are available. Among these are 10 arithmetic cams, mechanical models of similar triangles, and various types of bar-linkages<sup>(2)</sup>. These devices, while slightly faster than the integrator method, are also strictly limited as to frequency, and have a probable error of from 0.1 to 1 percent.

Introduction

The purpose of this study is to investigate the effects of various factors on the growth of the plant species studied. The study was conducted over a period of six months, during which time the plants were grown under different conditions. The results of the study are presented in the following sections.

The first section discusses the methods used in the study, including the selection of plant species, the experimental design, and the data collection procedures. The second section presents the results of the study, showing the growth of the plants under different conditions. The third section discusses the implications of the results, and the fourth section provides a conclusion.

The results of the study show that the growth of the plant species studied is significantly affected by the factors investigated. The implications of these results are discussed in the following section.

Also in the low frequency range, combination mechanical and electrical methods, utilizing servo driven bridges and potentiometers, have been used<sup>(3)</sup>. These devices have an error of about 0.1 percent.

An electronic pulse method has been used in which one input controls the amplitude of a rectangular wave, and the other input controls the duty ratio<sup>(4)</sup>. The time integral of the wave is proportional to the product of the two inputs. This scheme yields an error of less than 1 percent, but is limited in frequency to about 60 cps.

In a medium frequency range, up to about 1 kc, the simple electrodynamicometer and a probability method have been used. The electrodynamicometer operates on the principle that when two fluxes surrounding a fixed and movable coil are each made proportional to an input voltage, the resultant torque is proportional to the product of the two input voltages. The probability method, devised by Hardy<sup>(3)</sup>, is based on the fact that the probability of time coincidence of pulses occurring at noncommensurable rates is proportional to the products of the probabilities of the occurrence of the separate pulses at a given time. The input quantities control the duty ratios of the various pulse sources, while the duty ratio of the output of a coincidence circuit gives a measure of the product of the quantities. An





error of less than 4 percent has been obtained by this method.

Various purely electronic devices may be used to perform multiplication for frequencies up to about 50 kc.

Selected high-vacuum tubes with square-law characteristics have been utilized in this range<sup>(5)</sup>. The two voltages,  $e_1$  and  $e_2$ , to be multiplied are added and fed into the square-law tube. This yields  $e_1^2 + 2e_1e_2 + e_2^2$ . Simultaneously the two voltages are subtracted and fed into another square-law tube which yields  $e_1^2 - 2e_1e_2 + e_2^2$ . By subtracting the second of these results from the first, a product term,  $4e_1e_2$ , is obtained. Obviously any variation from perfect square-law characteristics in the tubes will cause error terms to arise. Balancing circuits have been designed to minimize such variations caused by aging, temperature changes and drift due to random fluctuations of the tube emission<sup>(6)</sup>. In this manner, errors of less than 0.5 percent have been obtained. A disadvantage of this type circuit is the difficulty of separating the product (including the d-c component of the product) from the plate supply voltage.

The use of germanium crystal rectifiers in a voltage range where they have approximately square-law characteristics largely overcomes drift difficulties. An additional advantage is that the input capacitance of these rectifiers is about one tenth that of the square-law

1. The first part of the document discusses the importance of maintaining accurate records of all transactions and activities. It emphasizes that this is essential for ensuring transparency and accountability in the organization's operations.

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tubes. However, the input transformer required to form the sum and difference of the inputs must also step down the input impedance to match the low impedance of the crystals. The product is then of very low voltage but does not require separation from the plate supply voltage, as was the case for the vacuum tubes. These rectifiers also exhibit hysteresis effects, and are not readily interchangeable because of large variations from the manufacturer's characteristics. Thus this use of crystal rectifiers shows little advantage over a similar use of vacuum tubes.

Exponential-law devices which use transfer characteristics of remote cut-off pentodes, current-voltage characteristics of high vacuum diodes in the negative plate voltage region, and copper-oxide or germanium rectifier bridges have been investigated by others<sup>(3,7)</sup>. These devices have been designed with an error of less than 2 percent, but they exhibit the same difficulties as the square-law devices previously discussed. An exponential-law method utilizing the principle of discharging capacitors has also been investigated<sup>(3)</sup>. The accuracy was about four times better than the types mentioned above. However, this device is limited in frequency to about 2 kc due to the time required for discharge of the capacitors.



A carrier-frequency multiplier has also been devised which makes use of a variable gain amplifier, the gain of which is controlled in accordance with one input signal by means of a feedback loop<sup>(3)</sup>. The amplifier input is the other signal. Thus the output is proportional to the product of the two inputs. This multiplier has an upper frequency limit of about 50 kc. It is the fastest device mentioned.

Multiplication methods which use multi-electrode vacuum tubes have utilized<sup>(9)</sup>. The converter tube has the property that its plate current is proportional to the product of the input voltages applied to the two control grids. The inputs must be unidirectional, therefore alternating inputs must be suitably biased. The important disadvantage of the converter tube as a multiplier is the difficulty of separating the desired product from the plate supply circuit. This difficulty can be overcome by using a tuned transformer, provided that one of the inputs is modulated prior to its application to the multiplier. The product is now shifted in frequency by the amount of the carrier and is readily amplified in a a-c coupled amplifier. The sense (positive or negative) of the product is determined by its phase with respect to the carrier.

The input applied via the modulator appears in the plate circuit and passes through the plate-circuit tuned

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transformer along with the product term. This input term is undesired, and although it lies outside the frequency range of the desired product term, it may be much larger in amplitude. The output of the desired term may therefore be limited by overloading due to this undesired term.

If both of the inputs were modulated, the product can be selected by means of a filter tuned to the sum or difference frequency of the two carrier input frequencies. There is then no undesired term at a high level in the output of the multiplier. Overloading and limitation of the output by the undesired term is thus avoided. The relative disadvantage of this scheme is the increased equipment required.

The example methods mentioned above far from complete the many ways in which electronic signal multiplication has been accomplished. They do, however, indicate the scope of the problem.

It can be shown that any nonlinear device, having an output which is a function of two variables, will exhibit in its output a sum or difference frequency term, a portion of which is proportional to the product of the two variables. The rest of that term can be considered as an error. If the error is small enough, multiplication can be nicely accomplished using a modulation scheme similar to that employed with the converter tube described





above. Bowers<sup>(10)</sup>, working with a transistor, has isolated this term. He has found that over a very limited dynamic range, the error portion is relatively small. In order to find out how well a transistor can accomplish the function of a converter tube, this investigation has set out to determine this error portion, and to attempt to find means of minimizing it.



## II. PROCEDURE

In this section the essential steps followed to obtain the final useful data are described. Details of these steps are presented in the Appendix, pages 32 to 42. Also presented in the Appendix are the investigations carried out, but later discontinued because they led either to inconclusive or inadequate information.

In all the following work, the symbolism as indicated below has been used: lower-case letters and subscripts for instantaneous values, upper-case letters for quantities of time-averaged values, upper-case letters with subscript "a" for peak values of sinusoidal varying quantities and upper-case letters with subscript "b" for direct values.

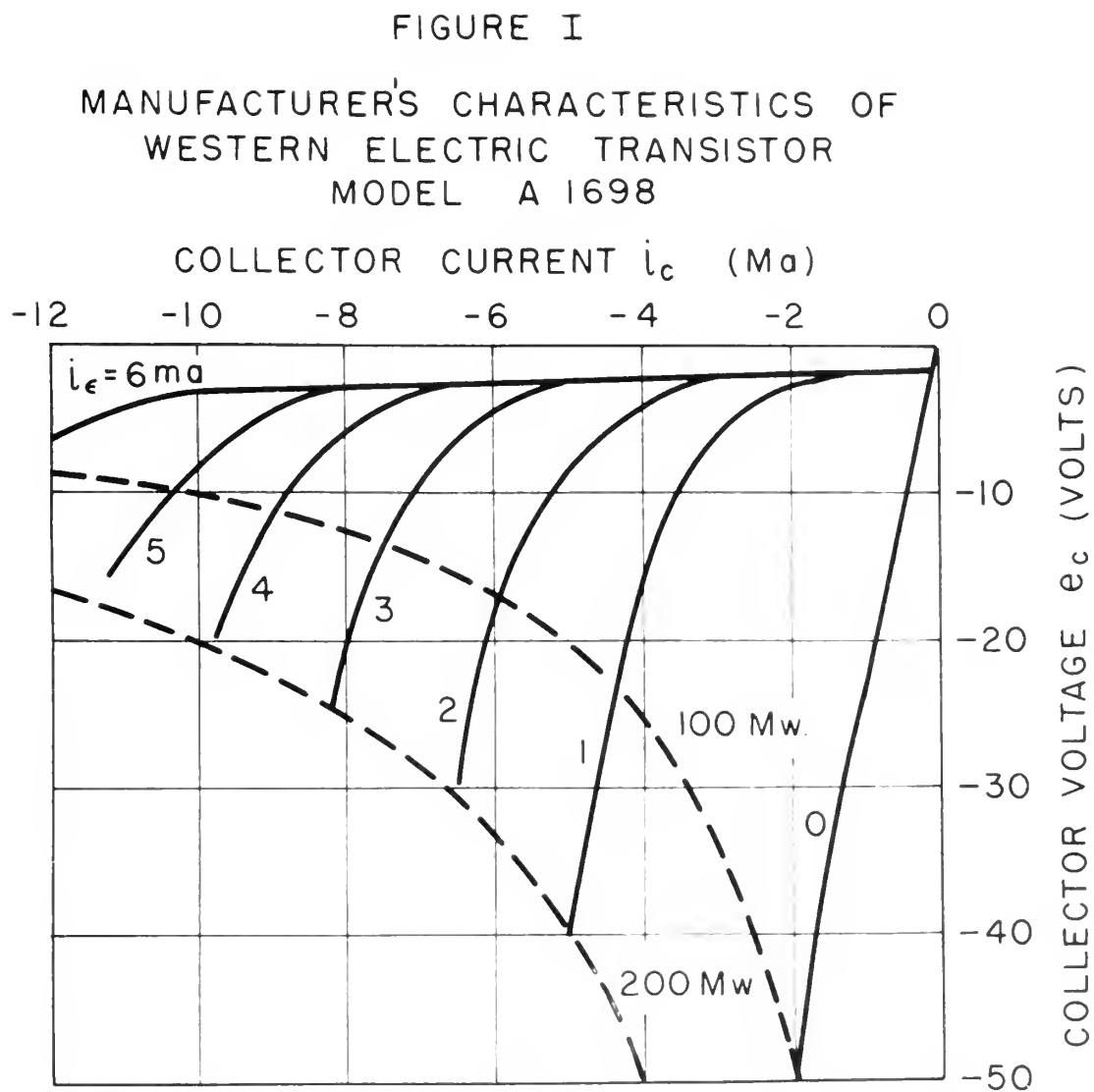
Two investigations were carried out simultaneously. In one study the application of a commercial General Electric, Type A-136, transistor to multiplication was investigated. In the other study a selected General Electric, Type 11, transistor was investigated along the same lines.

A typical set of collector current ( $i_c$ ) versus collector voltage ( $V_c$ ) characteristics are shown in Figure 1. From an examination of the collector characteristics it was decided that the most feasible type of multiplication would be that of multiplying collector

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voltage times emitter current, and taking collector current as the output. Since the transformation of currents to voltages and vice versa is a simple operation, the end result of multiplying voltages and getting a voltage proportional to the product is still retained.

It follows that for ideal multiplication, the collector characteristics should be such that the curves of constant emitter current ( $i_E$ ) are radial straight lines extending from the origin. Also, at constant collector voltage, the incremental values of collector current for equal incremental values of emitter current are constant. To show this:

$$\text{If } i_C = k \cdot i_E$$

$$\text{For } i_E = c_1$$

$$i_C = k \cdot c_1$$

This shows that the constant  $i_E$  curves are straight lines from the origin.

$$\text{For } i_C = c_2$$

$$i_E = \frac{1}{k} \cdot c_2$$

This shows that increments of collector current are constant for equal increments of emitter current.

For any general characteristics, the ratio can be visualized from inspection of a collector series expansion of  $i_C$  as a function of  $i_E$  and  $V_{CE}$  about an operating point. This complete expansion is shown in the Appendix, pages 38 to 42, for two sinusoidal inputs of different frequencies.

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It is seen from this expansion that the error signal in the sum and difference of the input frequencies has a component that is proportional to the product of the amplitudes of the inputs. But it also contains components which are proportional to other functions of the inputs; these are error components.

Mathematically:

$$i_c \text{ (sum or difference frequency)} = a_{10} i_e + a_{12} i_e^3 + a_{14} i_e^5 + a_{22} i_e^4 + a_{23} i_e^3 + a_{24} i_e^2 + \dots$$

It was found that, within the accuracy of the measurement devices employed, only the first three terms of the above equation were of sufficient magnitude to be measurable. If the fourth and subsequent terms are ignored, this equation becomes:

$$i_c \text{ (sum or difference frequency)} = a_{10} i_e + a_{12} i_e^3 + a_{14} i_e^5 \quad (1)$$

where

$$a_{10} = \left. \frac{\partial^2 i_c}{\partial i_e \partial e_c} \right|_{i_e=0}$$

$$a_{12} = \left. \frac{\partial^3 i_c}{\partial i_e^3} \times \frac{1}{6} \right|_{i_e=0}$$

$$a_{14} = \left. \frac{\partial^5 i_c}{\partial i_e^5} \times \frac{1}{120} \right|_{i_e=0}$$

For convenience the dimensions of the quantities used are in volts and milliamperes.

$$+ \frac{1}{3} \cdot 25 + \frac{1}{3} \cdot 16 + \frac{1}{3} \cdot 9 + \frac{1}{3} \cdot 4 + \frac{1}{3} \cdot 1 + \frac{1}{3} \cdot 0 + \dots + \frac{1}{3} \cdot 1 + \frac{1}{3} \cdot 0 + \dots$$

$$3^0 \cdot 1 + 3^1 \cdot 1 + 3^2 \cdot 1 + \dots = 1$$

$$\left[ \frac{1}{b}, \frac{1}{b} \right]$$

$$\left[ \frac{1}{b}, \frac{1}{b} \right]$$

$$\left[ \frac{1}{b}, \frac{1}{b} \right]$$

and the solutions are given by the following formulae:

Note that the complete expression, in the Appendix on page 41, indicates that for ideal multiplication,  $a_1$ ,  $a_3$  and all the following coefficients must be zero, thereby yielding the straight line characteristics described above. If that portion of the output ( $i_o$ ) that is at the sum or difference frequency of the inputs ( $\omega_1$  and  $\omega_2$ ) is isolated, perfect incremental multiplication would result if  $a_{12}$ ,  $a_{14}$ , and successive coefficients contributing to the sum or difference frequency portion vanished. Therefore one desired result was determination of  $a_1$ ,  $a_{12}$  and  $a_{14}$ . A second desired result was a means of maximizing  $a_1$  and minimizing  $a_{12}$  and  $a_{14}$ .

As part of our investigation, the point-contact pressure of a General Electric, Type 11, transistor was reduced. This had the fortuitous result of measurably straightening the constant  $i_b$  curves shown in the collector characteristics. It was further known that adjustments of the loading resistors shown in Figure 11 would have marked effects upon the characteristics of the equivalent transistor, where the equivalent transistor is now considered to be that inside the terminals.

Interdependent adjustments of  $R_1$ ,  $R_2$  and  $R_3$  were made until the characteristics of the equivalent transistor, about the operating point  $V_{ce} = -4$  volts,  $i_{ce} = 1.5$  ma, were apparently the most obtainable linear and parallel linearity and equal horizontal spacing.



The two investigations measured the coefficients  $a_{12}$ ,  $a_{14}$  and  $a_{11}$  by means of the circuits shown in Figures III and IV. For one investigation the whole of Figure III was inserted within the four terminals shown in Figure III. For the other, Figure IV was used as shown. The  $I_c$  and  $I_e$  inputs were varied and the  $I_c$  output was measured at  $\omega_2 - \omega_1 = 100$  cps, at  $2\omega_1 + \omega_2 = 1500$  cps, and at  $3\omega_2 - \omega_1 = 1000$  cps. Measurements at the local oscillator frequencies were used to calculate the magnitudes of  $a_{12}$  and  $a_{11}$ . The signs of these coefficients were determined from the consideration of the change in slope of the output, with one input constant and increasing the other. Sample calculations are shown in the Appendix, pages 43.

The expression thus obtained for the output current was compared with the measured values for its full accuracy. The dynamic range of multiplication was determined for a maximum error of 1 percent.

The range of possible input frequencies of the modified, parallel resonant circuit, was investigated by means of the circuit of Figure III, with constant the other of magnitudes of the input frequencies. The multiplier output was measured as a function of the sum of the input frequencies. The frequency of multiplication was determined for these different frequencies.

The first part of the paper is devoted to the study of the properties of the function  $f(x)$  defined by the equation  $f(x) = \int_0^x f(t) dt$ . It is shown that  $f(x)$  is a continuous function and that it satisfies the differential equation  $f'(x) = f(x)$ . The second part of the paper is devoted to the study of the properties of the function  $g(x)$  defined by the equation  $g(x) = \int_0^x g(t) dt$ . It is shown that  $g(x)$  is a continuous function and that it satisfies the differential equation  $g'(x) = g(x)$ . The third part of the paper is devoted to the study of the properties of the function  $h(x)$  defined by the equation  $h(x) = \int_0^x h(t) dt$ . It is shown that  $h(x)$  is a continuous function and that it satisfies the differential equation  $h'(x) = h(x)$ . The fourth part of the paper is devoted to the study of the properties of the function  $k(x)$  defined by the equation  $k(x) = \int_0^x k(t) dt$ . It is shown that  $k(x)$  is a continuous function and that it satisfies the differential equation  $k'(x) = k(x)$ . The fifth part of the paper is devoted to the study of the properties of the function  $l(x)$  defined by the equation  $l(x) = \int_0^x l(t) dt$ . It is shown that  $l(x)$  is a continuous function and that it satisfies the differential equation  $l'(x) = l(x)$ . The sixth part of the paper is devoted to the study of the properties of the function  $m(x)$  defined by the equation  $m(x) = \int_0^x m(t) dt$ . It is shown that  $m(x)$  is a continuous function and that it satisfies the differential equation  $m'(x) = m(x)$ . The seventh part of the paper is devoted to the study of the properties of the function  $n(x)$  defined by the equation  $n(x) = \int_0^x n(t) dt$ . It is shown that  $n(x)$  is a continuous function and that it satisfies the differential equation  $n'(x) = n(x)$ . The eighth part of the paper is devoted to the study of the properties of the function  $o(x)$  defined by the equation  $o(x) = \int_0^x o(t) dt$ . It is shown that  $o(x)$  is a continuous function and that it satisfies the differential equation  $o'(x) = o(x)$ . The ninth part of the paper is devoted to the study of the properties of the function  $p(x)$  defined by the equation  $p(x) = \int_0^x p(t) dt$ . It is shown that  $p(x)$  is a continuous function and that it satisfies the differential equation  $p'(x) = p(x)$ . The tenth part of the paper is devoted to the study of the properties of the function  $q(x)$  defined by the equation  $q(x) = \int_0^x q(t) dt$ . It is shown that  $q(x)$  is a continuous function and that it satisfies the differential equation  $q'(x) = q(x)$ .

FIGURE II  
EQUIVALENT TRANSISTOR

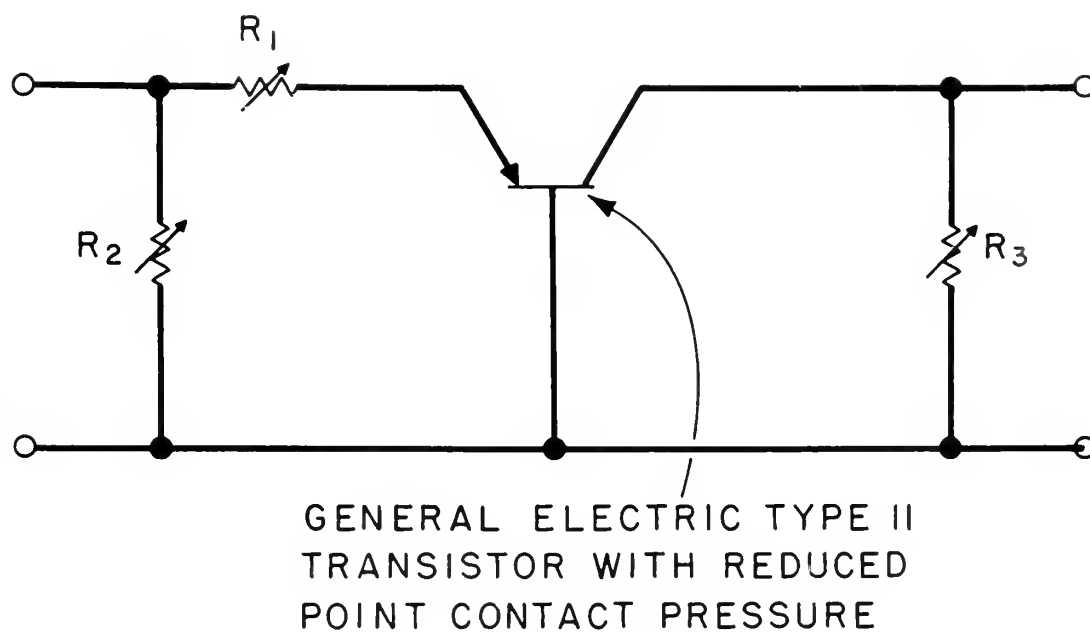
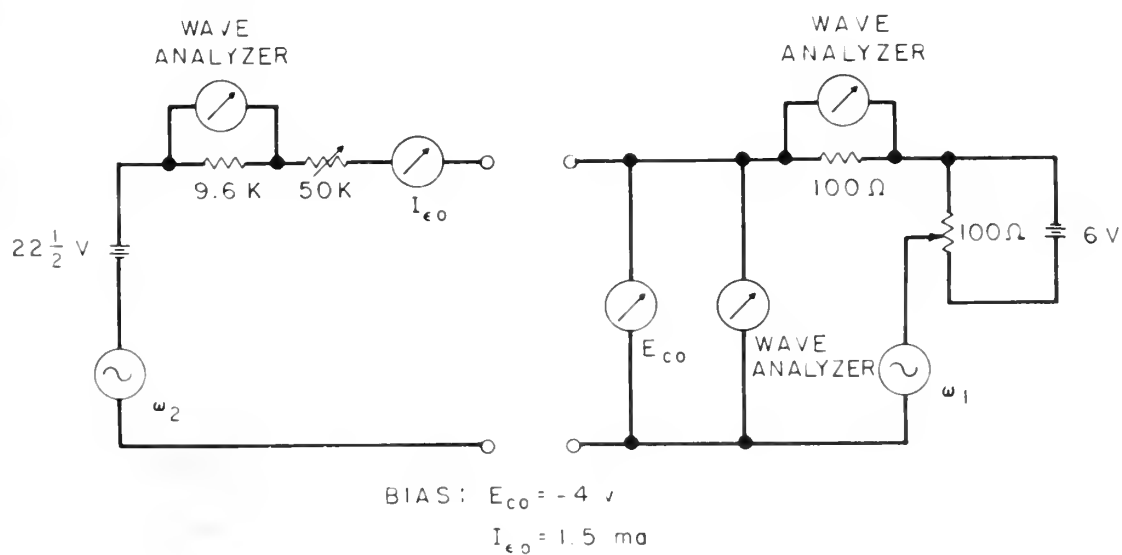


FIGURE III  
MEASURING CIRCUIT FOR DIFFERENCE FREQUENCY CURRENT







### III. RESULTS

#### Type A-1658 Investigation

The difference frequency component of the output ( $i_c$ ) was measured while using a standard, unpadding Western Electric, Type A-1658, transistor in the circuit of Figure IV. The observed data is plotted on Figures V and VI. Indicated on Figure V is the area of operation within which the maximum error of the output from perfect multiplication was less than 5 percent. The coefficients of equation (1) on page 11 were calculated yielding the following result:

$$i_c \text{ (sin or difference frequency)} = (4.31 - 0.0377 \frac{v_c}{V} - 0.0004 \frac{v_c^2}{V^2}) I_E \quad (2)$$

This equation defines the measured data out to  $v_c = 1.5$  volts within an average of 1 percent.

#### Adjustment of Resistive Gain with Type 11

The changes in the collector characteristics of an equivalent transistor as resistances in the resistances shown in Figure II were investigated.

$$\frac{R_1}{1}$$

Decreasing  $R_1$  moved the curves of constant  $i_c$  clockwise. The movement was not uniform. The  $i_c = 0$  curve remained relatively stationary as did the high  $i_c$  curves which were nearly horizontal. This adjustment had little

Section 1

1. The first part of the document discusses the importance of maintaining accurate records of all transactions. It emphasizes that every entry must be supported by a valid receipt or invoice. This ensures transparency and allows for easy verification of the data.

2. The second part of the document outlines the procedures for handling discrepancies. If a discrepancy is identified, the relevant parties must be notified immediately. A thorough investigation should be conducted to determine the cause of the error and to prevent it from recurring.

3. The third part of the document describes the process for updating the records. It is essential to ensure that all updates are made in a timely and accurate manner. This involves reviewing the existing records and making the necessary corrections.

4. The fourth part of the document discusses the importance of regular audits. Audits should be conducted at regular intervals to ensure that the records are accurate and up-to-date. This helps to identify any potential issues and allows for corrective action to be taken.

5. The fifth part of the document outlines the responsibilities of the personnel involved in the record-keeping process. It is the responsibility of all staff to ensure that the records are maintained accurately and that any changes are properly documented.

1 2 3 4 5 6 7 8 9 10 11 12

1. The first part of the document discusses the importance of maintaining accurate records of all transactions. It emphasizes that every entry must be supported by a valid receipt or invoice. This ensures transparency and allows for easy verification of the data.

2. The second part of the document outlines the procedures for handling discrepancies. If a discrepancy is identified, the relevant parties must be notified immediately. A thorough investigation should be conducted to determine the cause of the error and to prevent it from recurring.

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5. The fifth part of the document outlines the responsibilities of the personnel involved in the record-keeping process. It is the responsibility of all staff to ensure that the records are maintained accurately and that any changes are properly documented.

FIGURE IV  
MEASURING CIRCUIT FOR DIFFERENCE FREQUENCY CURRENTS

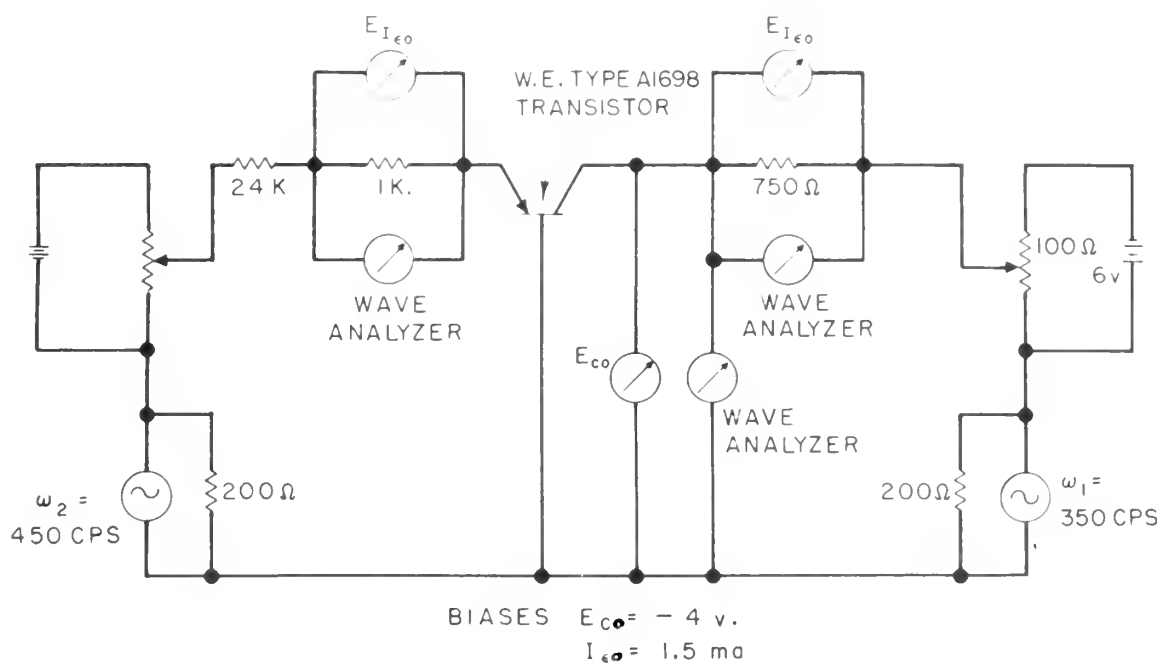




FIGURE V  
EXPERIMENTAL DIFFERENCE FREQUENCY OUTPUT  
CURRENT VS INPUTS APRIL 15, 1952 R. G. I.

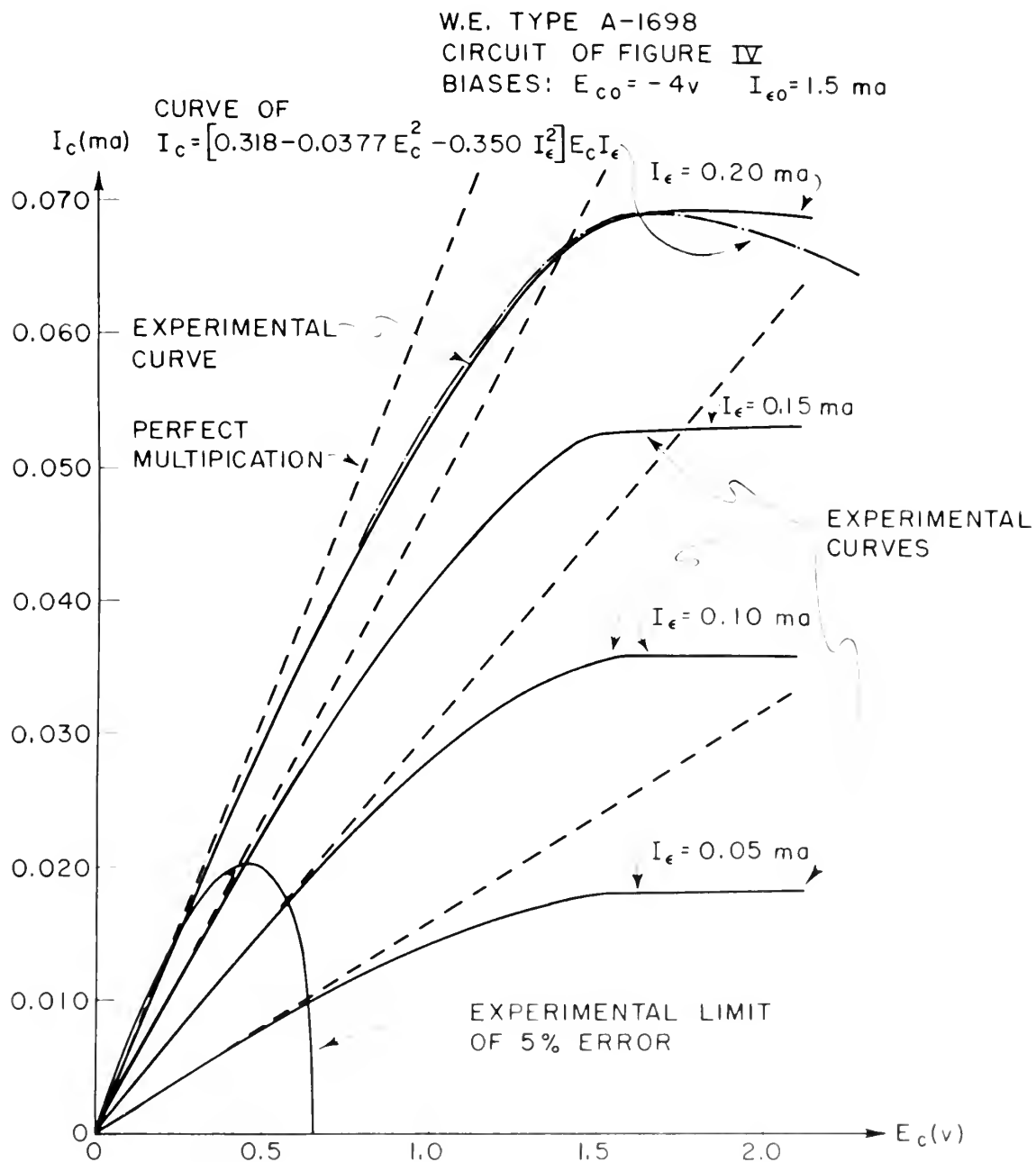
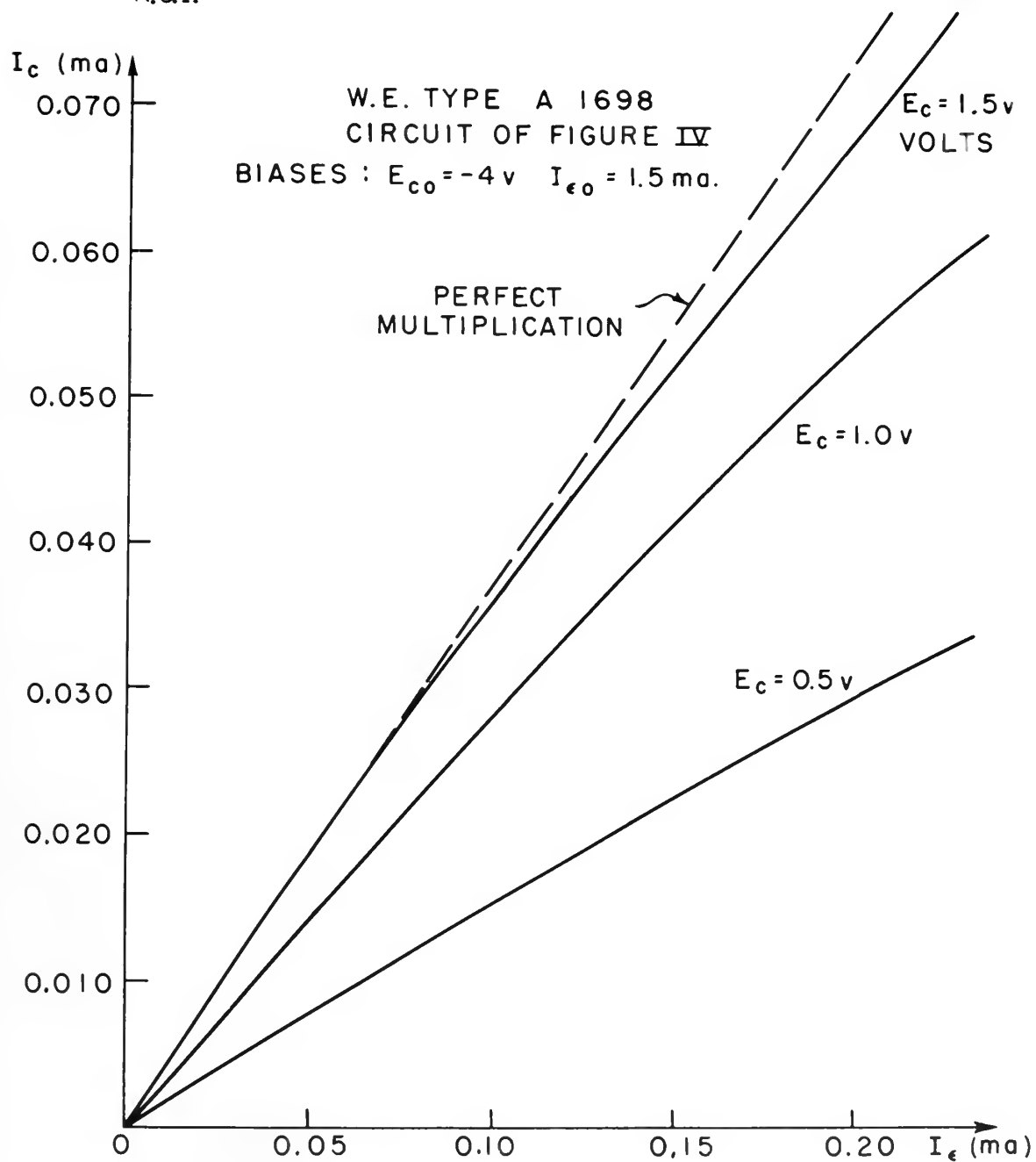




FIGURE VI

EXPERIMENTAL DIFFERENCE FREQUENCY OUTPUT  
CURRENT VS. INPUTS (CROSS CURVE) APRIL 15, 1952  
R.G.I.







effect upon the linearity, but the horizontal spacing was measurably changed.

### $R_2$

Decreasing  $R_2$  had a contrasting effect from that observed with a similar adjustment of  $R_1$ . The constant  $i_c$  curves were rotated counter-clockwise. As with  $R_1$ , the  $i_c = 0$  and high  $i_c$  curves remained relatively stationary. This adjustment also primarily effected the horizontal spacing.

### $R_3$

Decreasing  $R_3$  rotated all the constant  $i_c$  curves clockwise. Again the movement was not uniform. The higher  $i_c$  curves moved together more quickly than the lower curves. The linearity of the curves increased as  $R_3$  was decreased.

Optimum settings of these resistors were found to be approximately:  $R_1 = 100\Omega$ ,  $R_2 = 750\Omega$  and  $R_3 = 5,000\Omega$ .

### Low frequency investigation of circuit type 11

The general circuit, type 11, is a resistor with reduced point-contact pressure. It is used with the resistors of values determined above, as pictured in the circuit of Figure 11. The differential frequency component of the output ( $i_c$ ) has been measured and is plotted in Figure 11 and VIII. Collected data in Table II is the area of operation within which the maximum error of the output from perfect nullification was less than 1 percent. The calculations of equation (1) in type 11 were calculated within the

2

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7 8 9 10 11 12

13 14 15 16 17 18 19 20

21 22 23 24 25 26 27 28

29

FIGURE VII

EXPERIMENTAL DIFFERENCE FREQUENCY OUTPUT CURRENT VS INPUTS  
APRIL 4, 1952 S.N.R.

G. E. MODIFIED, PADDED TYPE II  
CIRCUIT OF FIGURE III

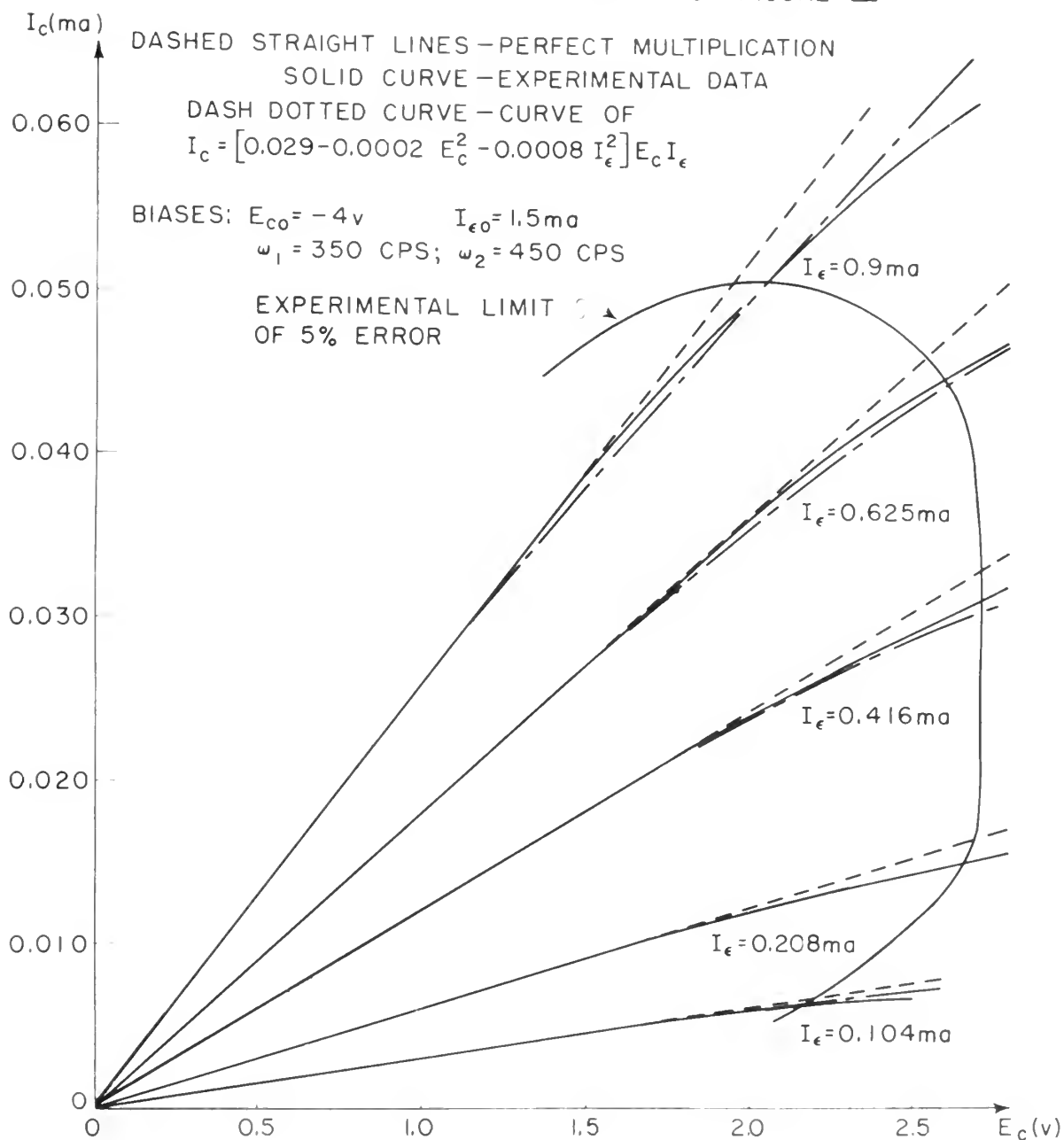
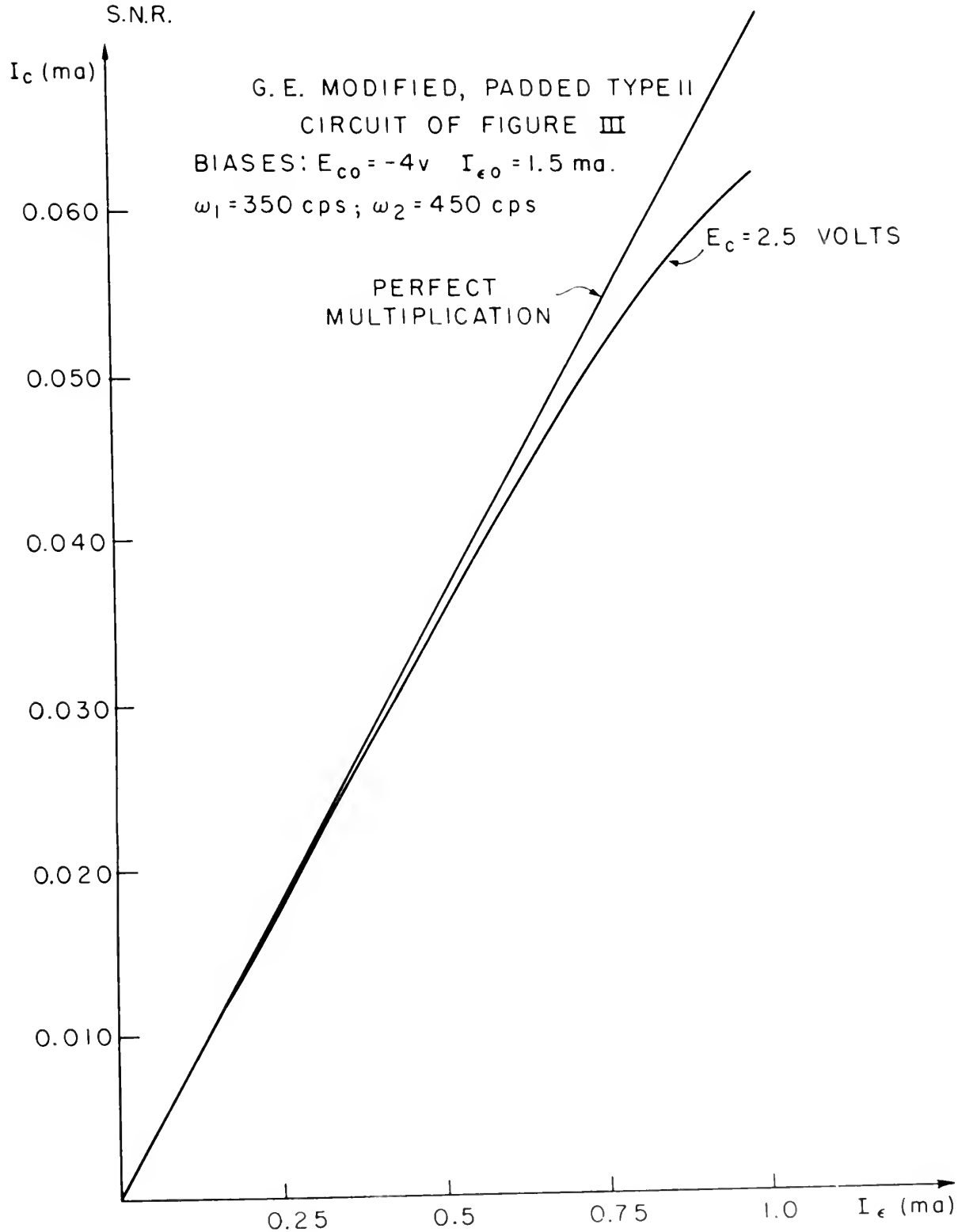




FIGURE VIII

EXPERIMENTAL DIFFERENCE FREQUENCY OUTPUT  
CURRENT VS. INPUTS (CROSS CURVE) APRIL 4, 1952  
S.N.R.





following result:

$$I_c \text{ (sum or difference frequency)} = (0.029 - 0.0004 \omega_c^2 - 0.0007 \omega_c) I_E \quad (3)$$

This equation defines the measured data within an accuracy of 3 percent.

#### Response as a function of frequency of the loaded Type 11

The values of  $\omega_1$  and  $\omega_2$  were varied in the circuit of Figure 11f. The sum or difference frequency component of the output ( $I_c$ ) was measured and is plotted on Figure 1X for  $I_E = 0.200$  ma, and on Figure 2 for  $I_E = 0.410$  ma. Figure 3f was derived from Figures 1X and 2 by choosing a constant  $I_c$  of 1 volt and plotting the difference frequency component of the output versus the sum of the input frequencies for the two output currents shown.

The coefficients of equation (1), as a function of frequency, were determined by trial and error to make this equation fit nearly all the data plotted on Figures 1X and 2. These coefficients are plotted on Figure 3f.

3

3

11

1. The first part of the paper is devoted to the study of the properties of the function  $f(x)$  defined by the equation

$$f(x) = \int_0^x \frac{1}{1+t^2} dt$$

It is well known that

$$f(x) = \arctan x$$

$$f'(x) = \frac{1}{1+x^2}$$

$$f(0) = 0$$

and hence

$$f(x) = \arctan x$$

for all  $x$ .

It is also known that

$$f(x) = \arctan x$$

$$f'(x) = \frac{1}{1+x^2}$$

and hence

$$f(x) = \arctan x$$

for all  $x$ .

$$f(x) = \arctan x$$



FIGURE IX  
EXPERIMENTAL DIFFERENCE FREQUENCY OUTPUT  
CURRENT VS. INPUTS APRIL 25, 1952 S.N.R., R.G.I.

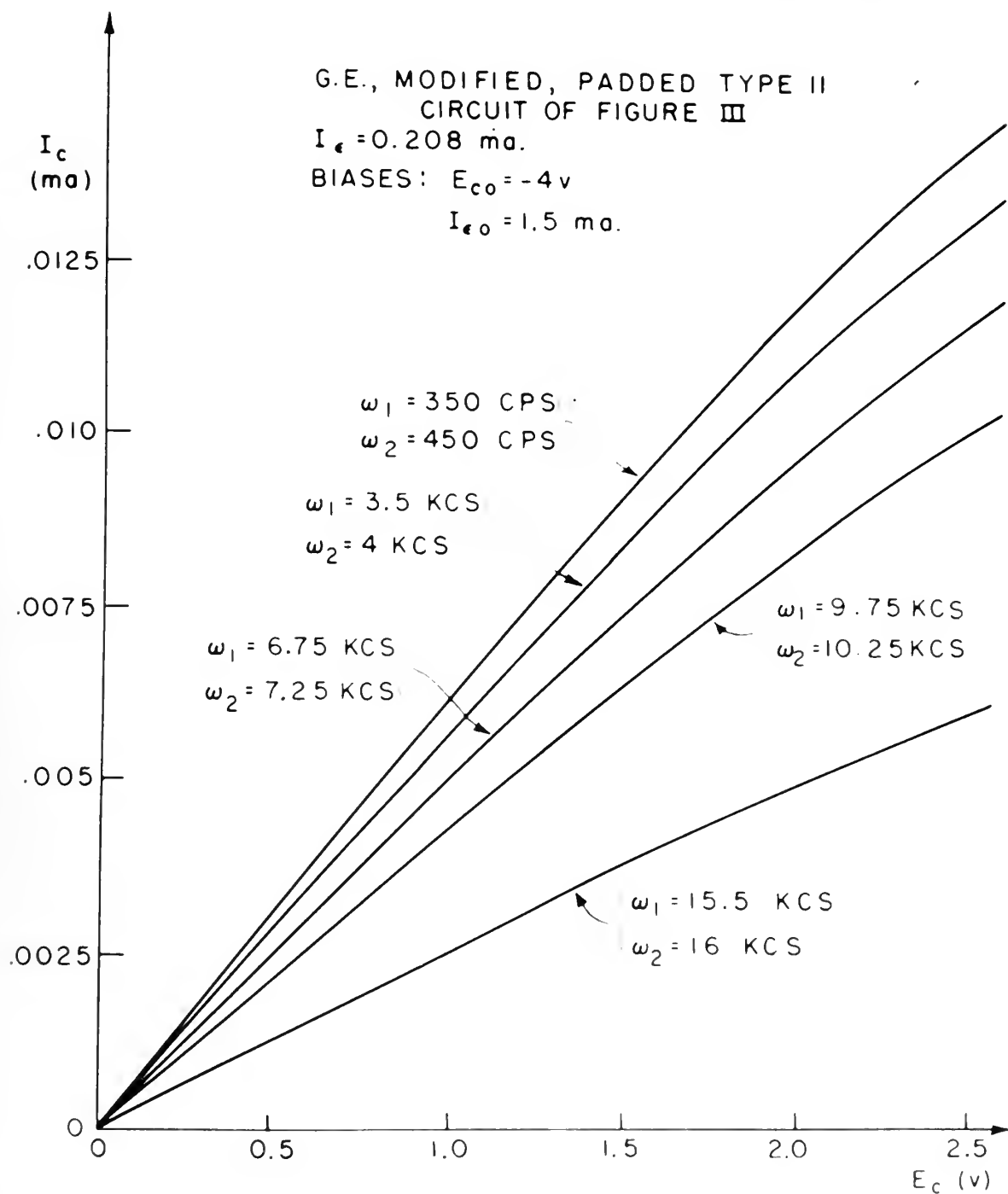
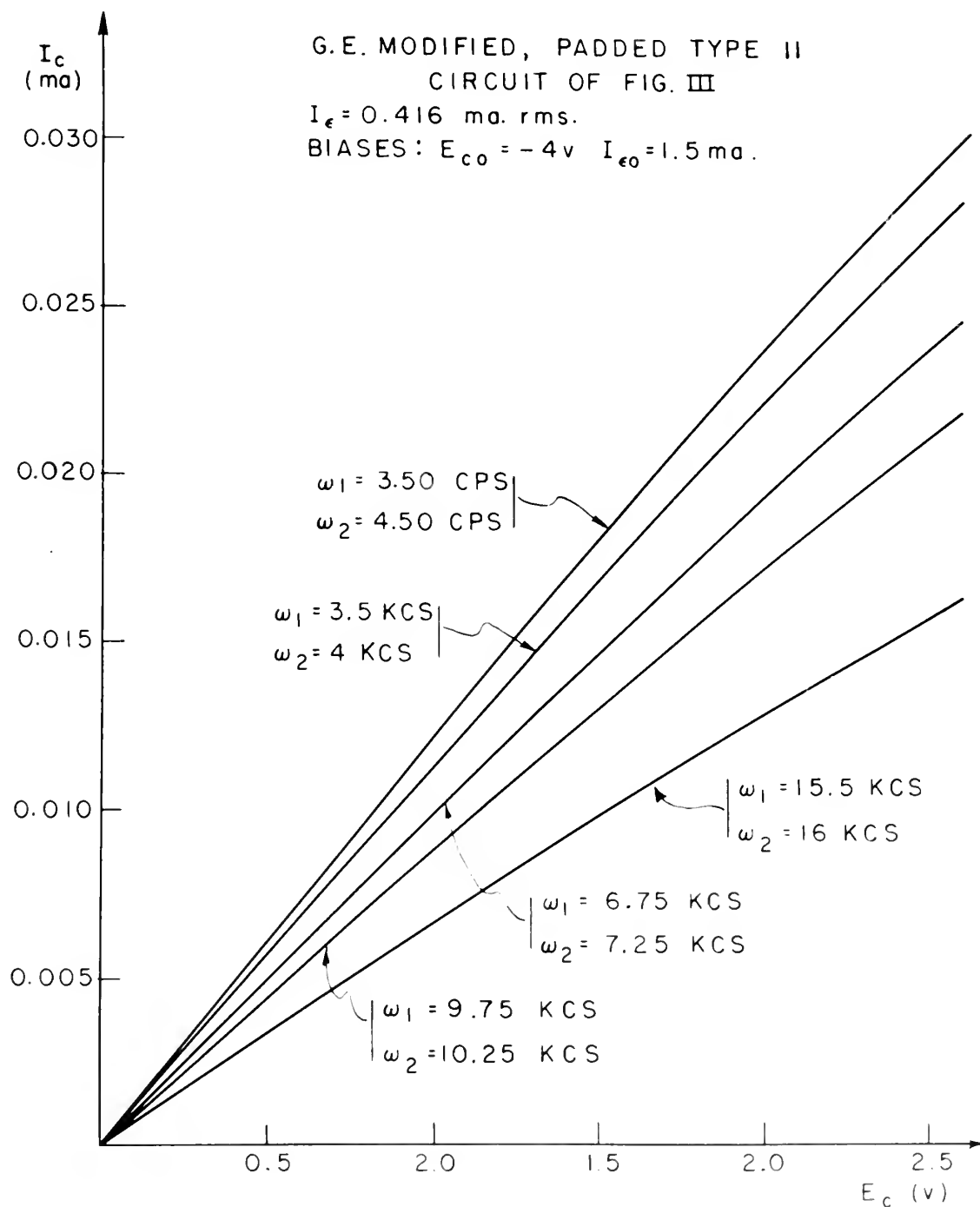




FIGURE X  
EXPERIMENTAL DIFFERENCE FREQUENCY OUTPUT  
CURRENT VS INPUTS APRIL 25, 1952 S.N.R. R.G.I.





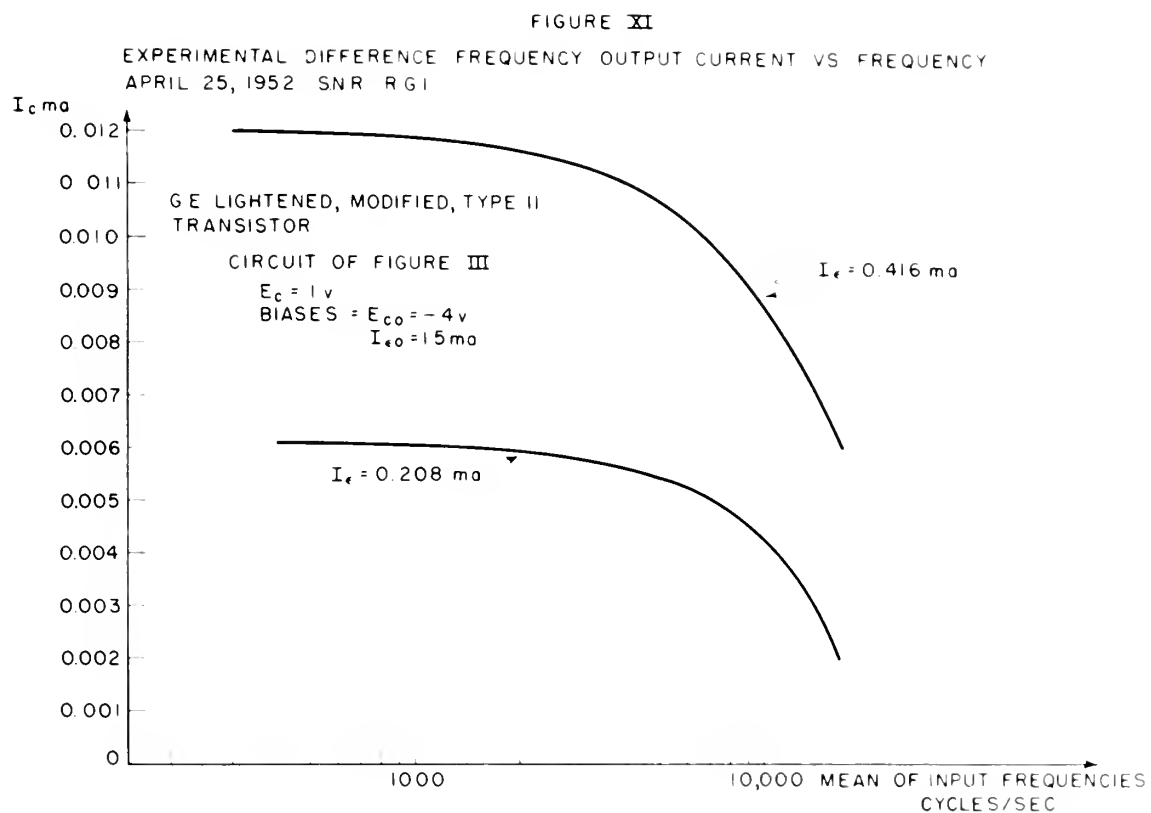
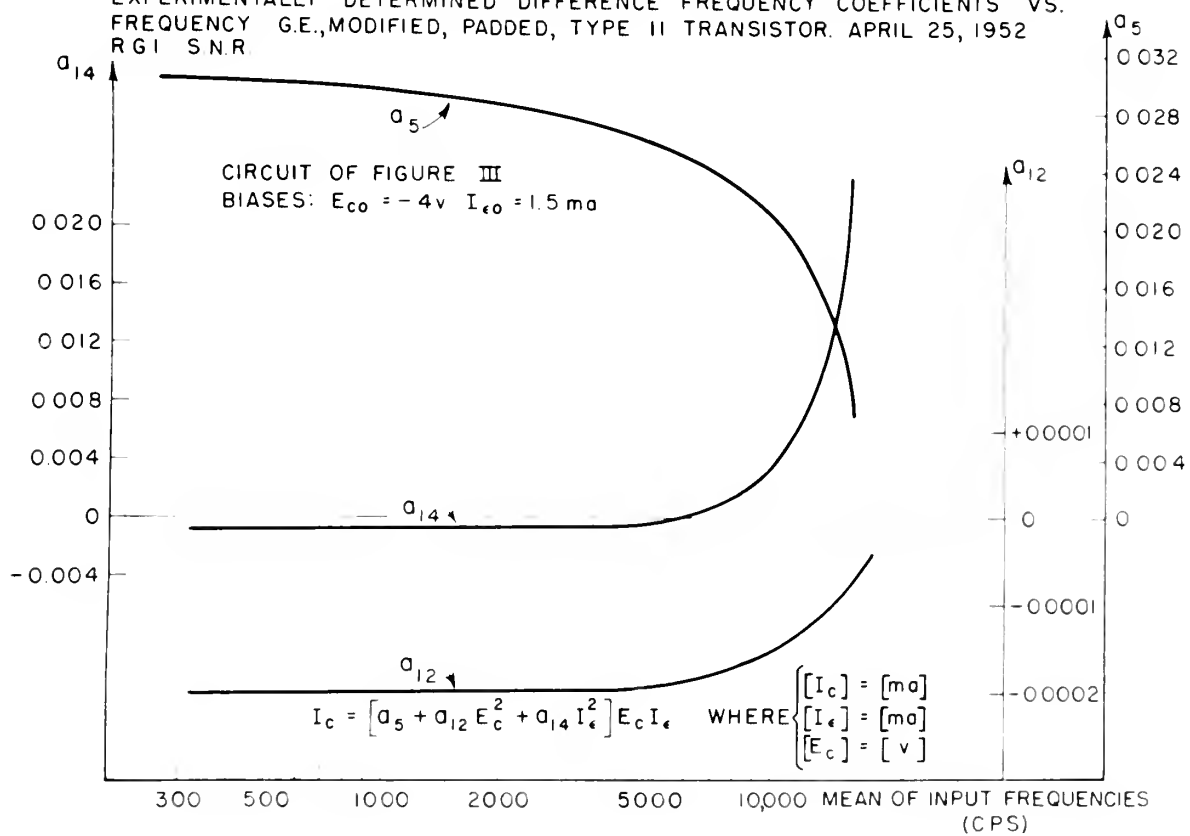




FIGURE XII

EXPERIMENTALLY DETERMINED DIFFERENCE FREQUENCY COEFFICIENTS VS.  
 FREQUENCY G.E., MODIFIED, PADDED, TYPE II TRANSISTOR. APRIL 25, 1952  
 RGI S.N.R.







*[Faint handwritten notes at the bottom of the page]*

### Optimum Adjustment of Operating Point

For comparison purposes the biases of the standard Western Electric, type 1-139, transistor were arbitrarily set at  $V_{CO} = -\frac{1}{2}$  v and  $I_{E0} = 1.5$  ma. It was felt that the transistor operation as a multiplier was not too sensitive to changes in the applied biases. However, since no investigation of various operating points has been made, it cannot be definitely concluded that the chosen operating point gives the maximum value or accuracy of multiplication. Because of this incision, it is recommended that further investigation be undertaken at different biases to determine the optimum operating point.

The operating point of the nonlinear resistances values used with the nonlinear circuit elements, type II, transistors were determined for the approximate maximum accuracy of multiplication. This procedure was discussed in detail in the preceding chapters 36 and 38. In this instance, it was felt that the error was not too great. It is conceivable, however, that this could be improved by corresponding maximum and minimum values; a suitable upper limit of the range of operation, and the lower limit to the range of noise. It is therefore felt that further investigation of nonlinear resistances and range of inputs should be considered. It could be accomplished

1. The first part of the report deals with the general situation in the country. It is a very interesting and informative study of the country's development since 1945. The author has done a great deal of research and has gathered a wealth of material. The report is well written and is a valuable contribution to the study of the country's development.

2. The second part of the report deals with the economic situation. It is a very interesting and informative study of the country's economic development since 1945. The author has done a great deal of research and has gathered a wealth of material. The report is well written and is a valuable contribution to the study of the country's economic development.

by first varying the applied biases, then adjusting the padding resistances as already described, and finally analyzing the difference frequency component of the output for range and accuracy.

#### Comparison between outputs of types 11 and 4-1693

For comparing the accuracy and range of multiplication between the two proposed transistor components, both the graphical presentation of Figures V and VII and the equations (2) and (3) will be utilized. For ready reference these equations are:

$$I_C = (0.313 - 0.0377 \frac{I_C^2}{I_E} - 0.258 \frac{I_E^2}{I_E}) I_C I_E \quad (2)$$

for the standard resistor loaded, type 4-1693, transistor,

$$\text{and } I_C = (0.029 - 0.0002 \frac{I_C^2}{I_E} - 0.0001 \frac{I_E^2}{I_E}) I_C I_E \quad (3)$$

for the padded, modified contact transistor, General Electric, Type 11, transistor.

Normalizing these equations with respect to the input product yields for equation (2)

$$\frac{I_C}{0.313} = (1 - 0.115 \frac{I_C^2}{I_E} - 1.1 \frac{I_E^2}{I_E}) I_C I_E,$$

and for equation (3)

$$\frac{I_C}{0.029} = (1 - 0.0009 \frac{I_C^2}{I_E} - 0.0001 \frac{I_E^2}{I_E}) I_C I_E.$$

From these normalized equations it is seen that for any given set of inputs, the output of the standard contact transistor is about 11 times as large as the output of the modified, padded contact transistor.

(1) The first part of the document is a letter from the President of the United States to the Congress, dated January 1, 1861. It is a very important document, as it contains the President's message to the Congress at the beginning of his first term. The letter is written in a very formal and dignified style, and it is one of the most important documents in the history of the United States.

(2) The second part of the document is a report from the Secretary of the Treasury, dated January 1, 1861. It is a very important document, as it contains the Secretary's report to the Congress on the state of the Treasury at the beginning of his first term. The report is written in a very formal and dignified style, and it is one of the most important documents in the history of the United States.

(3) The third part of the document is a report from the Secretary of the Interior, dated January 1, 1861. It is a very important document, as it contains the Secretary's report to the Congress on the state of the Interior at the beginning of his first term. The report is written in a very formal and dignified style, and it is one of the most important documents in the history of the United States.

(4) The fourth part of the document is a report from the Secretary of the War, dated January 1, 1861. It is a very important document, as it contains the Secretary's report to the Congress on the state of the War at the beginning of his first term. The report is written in a very formal and dignified style, and it is one of the most important documents in the history of the United States.

(5) The fifth part of the document is a report from the Secretary of the Navy, dated January 1, 1861. It is a very important document, as it contains the Secretary's report to the Congress on the state of the Navy at the beginning of his first term. The report is written in a very formal and dignified style, and it is one of the most important documents in the history of the United States.

(6) The sixth part of the document is a report from the Secretary of the State, dated January 1, 1861. It is a very important document, as it contains the Secretary's report to the Congress on the state of the State at the beginning of his first term. The report is written in a very formal and dignified style, and it is one of the most important documents in the history of the United States.

(7) The seventh part of the document is a report from the Secretary of the War, dated January 1, 1861. It is a very important document, as it contains the Secretary's report to the Congress on the state of the War at the beginning of his first term. The report is written in a very formal and dignified style, and it is one of the most important documents in the history of the United States.

(8) The eighth part of the document is a report from the Secretary of the Navy, dated January 1, 1861. It is a very important document, as it contains the Secretary's report to the Congress on the state of the Navy at the beginning of his first term. The report is written in a very formal and dignified style, and it is one of the most important documents in the history of the United States.

(9) The ninth part of the document is a report from the Secretary of the State, dated January 1, 1861. It is a very important document, as it contains the Secretary's report to the Congress on the state of the State at the beginning of his first term. The report is written in a very formal and dignified style, and it is one of the most important documents in the history of the United States.

(10) The tenth part of the document is a report from the Secretary of the War, dated January 1, 1861. It is a very important document, as it contains the Secretary's report to the Congress on the state of the War at the beginning of his first term. The report is written in a very formal and dignified style, and it is one of the most important documents in the history of the United States.

Of more importance however; the error portions of the output of the standard Western Electric transistor are 17.2 and 33.9 times as large as the corresponding error portions of the equivalent General Electric transistor.

Arbitrarily considering a maximum acceptable error of 5 percent, Figures V and VII show that the range of possible input values for the equivalent General Electric transistor is about four times that of the standard Western Electric transistor.

From these considerations it is evident, that for the chosen operating point, the modified, padded General Electric transistor offers greater possibilities for utilization in practical multiplication circuits.

#### Response as a Function of Frequency

The results of the frequency investigation of the modified, padded General Electric transistor are best analyzed from Figure XII. From an inspection of these curves it is concluded that as the mean of the input frequencies is increased, the accuracy falls off slightly until a frequency of about 4,000 cps is reached. At about this frequency accuracy starts to improve. The optimum of accuracy is reached at about 6,000 or 7,000 cps. Upon further increase in the mean of the input frequencies, the accuracy of multiplication is radically reduced. It is therefore concluded that accuracy gets an upper



limit on the input frequencies of about 10,000 cps.

This places a severe frequency limitation upon the possible use of this equivalent transistor as an electronic multiplying device.

#### Response as a Function of Temperature

In certain applications the frequency limitation may not be critical. Because of this, a further investigation of the effect of temperature variations upon the equivalent transistor response is recommended.

#### Interchangeability

Finally, an investigation is recommended to determine the interchangeability of transistors; that is, the variation of multiplicative characteristics upon several transistors of the same type.

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V. APPENDIX



## A. DETAILS OF PROCEDURE

### Preliminary Procedure

An attempt was initially made to check the transistor's adaptability to electronic multiplication by mathematical analysis of the usually accepted linear equivalent circuit. This approach proved useless since multiplication is essentially a nonlinear operation which obviously could not be derived from a linear equivalent circuit. An attempt was then made to determine the transistor's adaptability to the problem by a combination mathematical and graphical analysis of the transistor characteristics as published by the manufacturer. No definite conclusion could be reached by this method because of the infinite number of possible choices as to operating points and amplitudes of sinusoidal inputs. Further, the accuracy of this type of analysis was very poor due to inherent graphical inaccuracies. This became particularly apparent when small sinusoidal inputs were considered.

It was then decided to investigate the problem primarily by experimental methods and subsequently attempt to correlate the experimental findings with the transistor characteristics.

### Type A-1695 Investigation with Direct Inputs

An experimental investigation of the application of an

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1. The purpose of this document is to provide information regarding the activities of the [redacted] and the [redacted] in the [redacted] area. This information is being provided for your information and is not to be distributed outside of your office.

2. The [redacted] has been identified as a [redacted] and is currently active in the [redacted] area. The [redacted] has been identified as a [redacted] and is currently active in the [redacted] area. The [redacted] has been identified as a [redacted] and is currently active in the [redacted] area.

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unpaired Weston Electric, Type A-107, transistor was made. Direct current and voltage inputs were first used. The results were again inconclusive, probably because of measurement difficulties. The incremental d-c values to be measured were so small with respect to the bias values that accuracy of measurement was not possible even after balancing out a fixed portion of the bias.

Type A-107 Investigation with an Alternating Input

In an attempt to circumvent this difficulty the unpaired transistor was driven with a direct current current input and a small a-c signal collector voltage input. The a-c signal was not at all large enough to be isolated by blocking the d-c part with a capacitor whose impedance to the a-c signal was small. The result of this experiment was inconclusive. A properly biased a-c signal, of about 100 mV, 1000 Hz, within 1% react, to the input of the a-c collector inputs was obtained. The a-c signal was then between the true input and the output. The output was expected to be a straight line. It was by reference to a signal of 100 mV, 1000 Hz. The relation will result in,  $i_c = i_b + i_e$ , and the slopes of the constant  $i_c$  and  $i_e$  are proportional to the value of  $i_c$  and  $i_e$  respectively.  $i_c$  and  $i_e$  are straight lines.

1. The first part of the document is a list of names and addresses. The names are written in a cursive hand, and the addresses are written in a more formal, printed hand. The list is organized into two columns, with names on the left and addresses on the right.

2. The second part of the document is a list of names and addresses. The names are written in a cursive hand, and the addresses are written in a more formal, printed hand. The list is organized into two columns, with names on the left and addresses on the right.

3. The third part of the document is a list of names and addresses. The names are written in a cursive hand, and the addresses are written in a more formal, printed hand. The list is organized into two columns, with names on the left and addresses on the right.

4. The fourth part of the document is a list of names and addresses. The names are written in a cursive hand, and the addresses are written in a more formal, printed hand. The list is organized into two columns, with names on the left and addresses on the right.

5. The fifth part of the document is a list of names and addresses. The names are written in a cursive hand, and the addresses are written in a more formal, printed hand. The list is organized into two columns, with names on the left and addresses on the right.

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This investigation indicated that there was a restricted area over which these stipulations held within plus or minus 5 percent. Though this experiment was encouraging it provided a very limited solution to the general problem of multiplication. For this reason no further investigation from this viewpoint was attempted.

#### Resistive Loading of Type A-1680

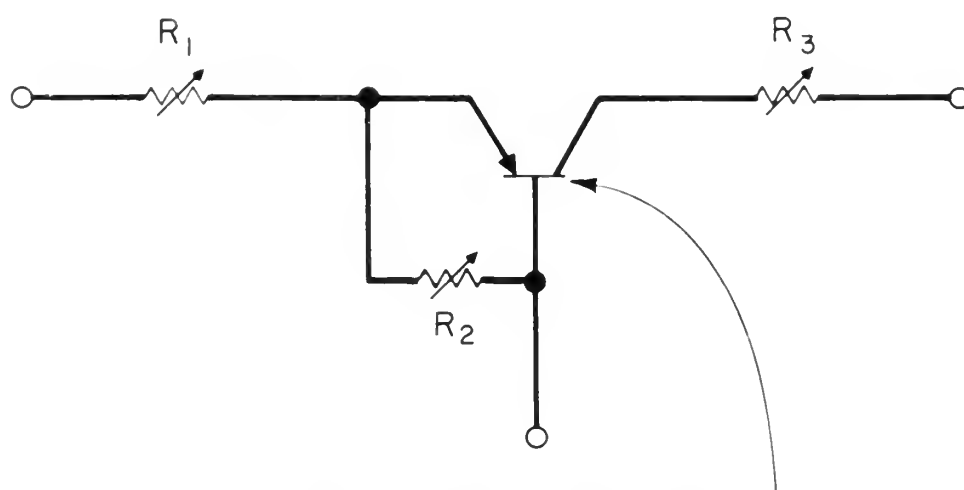
From a study of the desired characteristics as discussed on page 10 of the procedure it was decided to attempt to improve the standard transistor characteristics. To accomplish this the resistive network of Figure 2.11 was added to form another equivalent transistor. Through adjustment of the resistors  $R_1$ ,  $R_2$  and  $R_3$ , it was hoped to attain perfect multiplication over at least a limited range of inputs. Unfortunately this work on the germanium electric transistor, failed as shown in Figure 2.12, failed to produce the desired results. The linearity of the constant  $i_c$  curves could be improved, as the horizontal spacing of these curves tends to be nearly equal. However, the desired results could be accomplished only by a series resistance of 100 ohms. An attempt to improve the multiplication by the addition of padding resistors ( $R_1$ ,  $R_2$ , and  $R_3$ ) was not made at this juncture because it was realized that it would be more desirable to make a comparison between the multiplicative

1. The first part of the report is a general  
description of the project and its objectives.  
2. The second part is a detailed description of the  
methodology used in the study.  
3. The third part is a description of the results  
of the study.  
4. The fourth part is a discussion of the results  
and their implications.

5. The fifth part is a conclusion and a list of  
references.  
6. The sixth part is a list of appendices.  
7. The seventh part is a list of figures and  
tables.  
8. The eighth part is a list of abbreviations.  
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66. The sixty-sixth part is a list of appendices.  
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93. The ninety-third part is a list of symbols.  
94. The ninety-fourth part is a list of footnotes.  
95. The ninety-fifth part is a list of references.  
96. The ninety-sixth part is a list of appendices.  
97. The ninety-seventh part is a list of figures and  
tables.  
98. The ninety-eighth part is a list of abbreviations.  
99. The ninety-ninth part is a list of symbols.  
100. The hundredth part is a list of footnotes.



FIGURE XIII  
EQUIVALENT PADDED TRANSISTOR



WESTERN ELECTRIC  
TYPE A 1698 TRANSISTOR



ability of a standard transistor and that of one with reduced point-contact pressure and adjusted padding resistors.

#### Approximate Adjustment of Padding Resistors Shown in Figure II

The qualitative effects of changes in the values of the padding resistors shown in Figure II on page 14 was assessed by means of a plotter that automatically traced the collector characteristics. An approximate adjustment was made for optimum mutual linearity and equal horizontal spacing. These values were:  $R_1 = 2.0\Omega$ ,  $R_2 = 1.6\Omega$ ,  $R_3 = 1.9\Omega$ .

#### Static Characteristics

Through use of the circuit of Figure I, page 14, with the a-c generator short circuited, an attempt was made to measure the static collector characteristics for various values of the padding resistors. This proved impossible because the electric resistance depended upon the direction from which it was approached. This phenomenon was attributed primarily to contact effects. These effects had been investigated because the transistor had been exposed for 11.5 hours. A typical characteristic is plotted with varying  $I_E$  from zero to five milliamperes. It shows a 25 percent difference in the  $I_C$  readings between the first and second zero of  $I_E$ .

ORIGINAL ARTICLES

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### Linearity Check

To determine the effect on radial linearity due to changes in the padding resistors, the following scheme was first tried. At a given operating point,  $i_E$  was held constant and an alternating signal ( $e_o$ ) of known amplitude was applied. The alternating component of collector current ( $i_c$ ) was measured. If the constant  $i_E$  curves were to be linear, it follows that  $i_c$  should be proportional to  $e_o$ . Even though it had been observed on the plotter that changes in the padding resistors had marked effect on the linearity, it proved very difficult to discern these effects with this scheme.

A second method was tried which proved quite satisfactory. Emitter current was again held constant, but instead of keeping the operating point fixed and varying  $e_o$ ,  $e_o$  was held constant and the operating point was moved up and down the constant  $i_E$  curve. If the curves were to be linear, it follows that  $i_c$  should be constant. Around the operating point of  $V_{CE} = -10$  volts and  $i_E = 1.5$  ma, the optimum linearity is obtained with  $R_1 = 400\Omega$ ,  $R_2 = 10\Omega$ , and  $R_3 = 10\Omega$ .

### Horizontal Linearity Check

To check the horizontal linearity, an alternating signal ( $e_e$ ) was applied to the base. The operating point was held constant and  $i_c$  was measured.



alternating component of collector current was ensured. If the horizontal spacing was to be equal,  $i_c$  should be constant. A consideration from this point of view yielded optimum values of  $R_1 = 400\Omega$ ,  $R_2 = 750\Omega$ , and  $R_3 = 5\Omega$ .

Linearity with these values was checked. Since the linearity was not greatly different from that obtained with the optimum for linearity, whereas changes in the resistors had a critical effect on spacing, it was decided to use these last values for the succeeding work.

### Collector Characteristics

Sufficient data had been taken when determining spacing and linearity to plot the collector characteristics of the modified, point transistor. The information this is shown in Figure IV.

### Taylor Series Expansion

$$i_c = f(e_b, i_e)$$

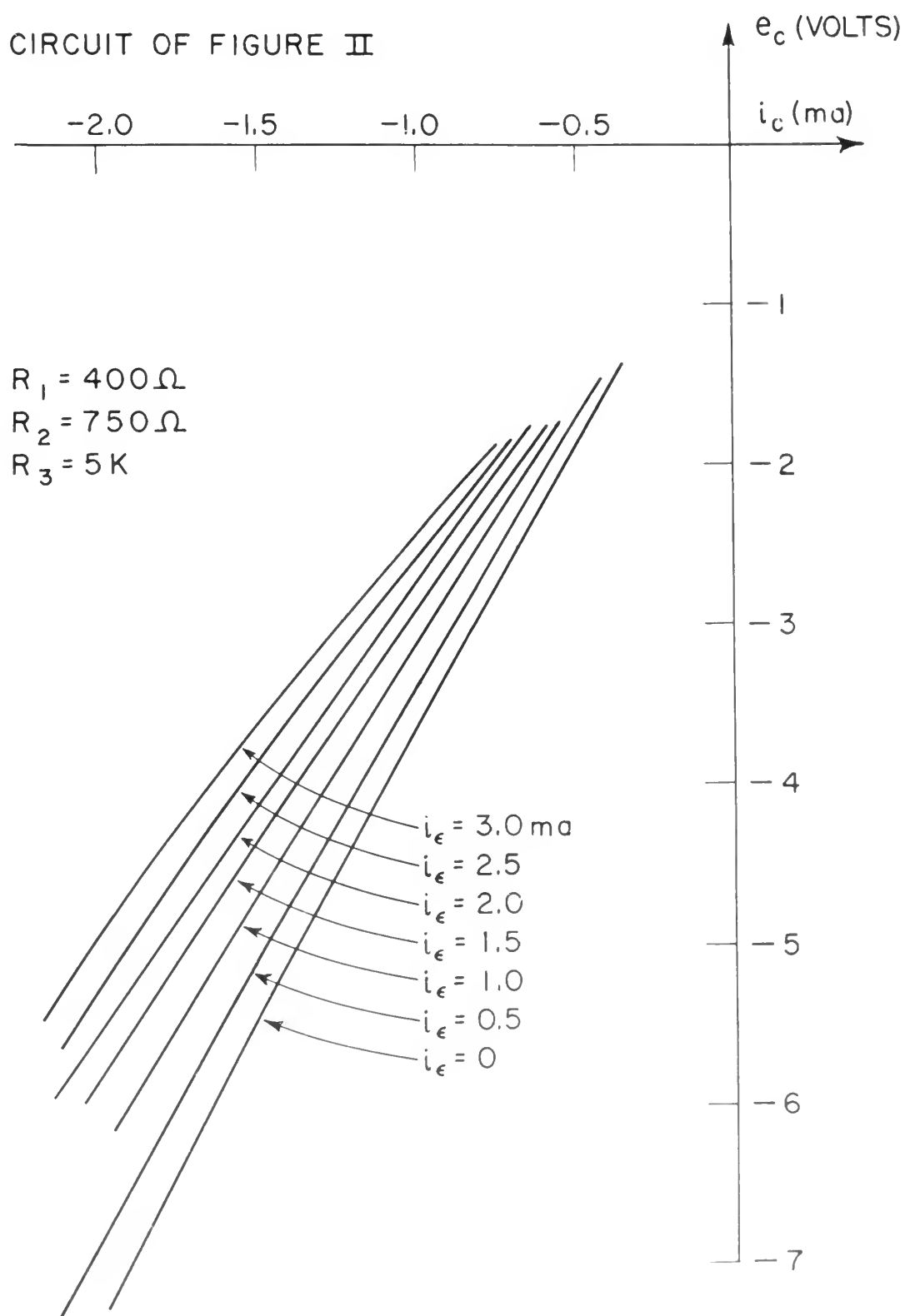
When expanded the Taylor series is

$$\left[ \frac{1}{(r-1)!} \left\{ (e_b - e_0) \frac{\partial^r}{\partial e_b^r} + (i_e - i_{e0}) \frac{\partial^r}{\partial i_e^r} \right\}^{r-1} f(e_b, i_e) \right]_{\substack{e_b = e_0 \\ i_e = i_{e0}}} \\ i_c = f(e_0, i_{e0}) + (e_b - e_0) \frac{\partial f}{\partial e_b} + (i_e - i_{e0}) \frac{\partial f}{\partial i_e} + \dots$$





FIGURE XIV  
COLLECTOR CHARACTERISTICS OF PADDED  
TRANSITOR MARCH 17, 1952 S. N. R.





$$\begin{aligned}
& \left[ (e_0 - e_0) \frac{\partial^2 f(\dots, \epsilon_0)}{\partial e_0^2} + 2(e_0 - e_0)(1_\epsilon - 1_{\epsilon_0}) \frac{\partial^2 f(\dots, 1_{\epsilon_0})}{\partial 1_\epsilon \partial e_0} + \right. \\
& \left. (1_\epsilon - 1_{\epsilon_0}) \frac{\partial^2 f(\dots, 1_{\epsilon_0})}{\partial 1_\epsilon^2} \right] + \\
& 1/3 \left[ (e_0 - e_0) \frac{\partial^3 f(\dots, \epsilon_0)}{\partial e_0^3} + 3(e_0 - e_0)(1_\epsilon - 1_{\epsilon_0}) \frac{\partial^3 f(\dots, \epsilon_0)}{\partial e_0^2 \partial 1_\epsilon} + \right. \\
& \frac{\partial^3 f(\dots, 1_{\epsilon_0})}{\partial e_0^2 \partial 1_\epsilon} + \frac{\partial^3 f(\dots, -\epsilon_0)}{\partial 1_\epsilon} + \\
& 3(e_0 - e_0)(1_\epsilon - 1_{\epsilon_0})^2 \frac{\partial^3 f(\dots, \epsilon_0)}{\partial e_0 \partial 1_\epsilon^2} + \frac{\partial^3 f(\dots, 1_{\epsilon_0})}{\partial 1_\epsilon^3} + \\
& \left. (1_\epsilon - 1_{\epsilon_0})^2 \frac{\partial^3 f(\dots, 1_{\epsilon_0})}{\partial e_0 \partial 1_\epsilon^2} \right] + \\
& 1/4 \left[ (e_0 - e_0) \frac{\partial^4 f(\dots, \epsilon_0)}{\partial e_0^4} + \right. \\
& 4(e_0 - e_0)(1_\epsilon - 1_{\epsilon_0}) \frac{\partial^4 f(\dots, \epsilon_0)}{\partial e_0^3 \partial 1_\epsilon} + \frac{\partial^4 f(\dots, \epsilon_0)}{\partial e_0^3 \partial 1_\epsilon} + \\
& 4(e_0 - e_0)(1_\epsilon - 1_{\epsilon_0}) \frac{\partial^4 f(\dots, \epsilon_0)}{\partial e_0^2 \partial 1_\epsilon^2} + \frac{\partial^4 f(\dots, \epsilon_0)}{\partial e_0^2 \partial 1_\epsilon^2} + \\
& 4(e_0 - e_0)(1_\epsilon - 1_{\epsilon_0})^2 \frac{\partial^4 f(\dots, \epsilon_0)}{\partial e_0 \partial 1_\epsilon^3} + \frac{\partial^4 f(\dots, \epsilon_0)}{\partial e_0 \partial 1_\epsilon^3} + \\
& \left. (1_\epsilon - 1_{\epsilon_0})^2 \frac{\partial^4 f(\dots, \epsilon_0)}{\partial 1_\epsilon^4} \right] + \dots
\end{aligned}$$

$$+ \frac{(a_1 + a_2 + \dots + a_n) \cdot b}{a_1 \cdot b + a_2 \cdot b + \dots + a_n \cdot b} = \frac{(a_1 + a_2 + \dots + a_n) \cdot b}{b(a_1 + a_2 + \dots + a_n)} = 1$$

$$+ \left[ \frac{a_1 + a_2 + \dots + a_n}{a_1 + a_2 + \dots + a_n} \cdot b \right]$$

$$(a_1 + a_2 + \dots + a_n) \cdot b = b(a_1 + a_2 + \dots + a_n)$$

$$+ \frac{a_1 + a_2 + \dots + a_n}{a_1 + a_2 + \dots + a_n} \cdot b = b$$

$$+ \frac{a_1 + a_2 + \dots + a_n}{a_1 + a_2 + \dots + a_n} \cdot b = b$$

$$+ \left[ \frac{a_1 + a_2 + \dots + a_n}{a_1 + a_2 + \dots + a_n} \cdot b \right]$$

$$+ \frac{a_1 + a_2 + \dots + a_n}{a_1 + a_2 + \dots + a_n} \cdot b = b$$

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$$+ \frac{a_1 + a_2 + \dots + a_n}{a_1 + a_2 + \dots + a_n} \cdot b = b$$

$$\dots + \left[ \frac{a_1 + a_2 + \dots + a_n}{a_1 + a_2 + \dots + a_n} \cdot b \right]$$

Since the partial derivatives at a particular operating point are constant, the notation may be simplified by writing them as  $b$ 's with subscripts. If the excursions from the operating point are sinusoidal we can write the following expressions:

$$e_0 - e_0 = e_m \sin \omega_1 t$$

$$i_e - i_{e0} = i_{em} \sin (\omega_2 t + \psi)$$

$$i_c = b_1 + b_2 e_m \sin \omega_1 t + b_3 i_{em} \sin (\omega_2 t + \psi)$$

$$+ \frac{1}{2} \left[ b_4 e_m^2 \sin^2 \omega_1 t + b_5 e_m i_{em} \sin \omega_1 t \sin (\omega_2 t + \psi) + b_6 i_{em}^2 \sin^2 (\omega_2 t + \psi) \right]$$

$$+ \frac{1}{6} \left[ b_7 e_m^3 \sin^3 \omega_1 t + 3 b_8 e_m^2 i_{em} \sin^2 \omega_1 t \sin (\omega_2 t + \psi) \right.$$

$$+ 3 b_9 e_m i_{em}^2 \sin \omega_1 t \sin^2 (\omega_2 t + \psi) + b_{10} i_{em}^3 \sin^3 (\omega_2 t + \psi) \left. \right]$$

$$+ \frac{1}{24} \left[ b_{11} e_m^4 \sin^4 \omega_1 t + b_{12} e_m^3 i_{em} \sin^3 \omega_1 t \sin (\omega_2 t + \psi) \right.$$

$$+ 6 b_{13} e_m^2 i_{em}^2 \sin^2 \omega_1 t \sin^2 (\omega_2 t + \psi) + b_{14} e_m i_{em}^3 \sin \omega_1 t \sin^3 (\omega_2 t + \psi) + b_{15} i_{em}^4 \sin^4 (\omega_2 t + \psi) \left. \right] + \dots$$

If the appropriate trigonometric substitutions are made, there is obtained:

$$i_c = b_1 + b_2 e_m \sin \omega_1 t + b_3 i_{em} \sin (\omega_2 t + \psi)$$

$$+ \frac{1}{2} \left[ b_4 e_m^2 (1 - \cos 2\omega_1 t) + 2 b_5 e_m i_{em} (-\cos \{(\omega_1 + \omega_2)t + \psi\}) \right.$$

$$+ \left. b_6 i_{em}^2 \cos \{(\omega_1 - \omega_2)t - \psi\} + b_7 e_m^3 (1 - \cos \{2\omega_1 t + \psi\}) \right]$$

$$+ \frac{1}{6} \left[ b_8 e_m^3 (1/4 \sin 3\omega_1 t - 3/4 \sin \omega_1 t) + 3 b_9 e_m^2 i_{em} \right.$$

$$\left. (\sin \{ \omega_2 t + \psi \} - \sin \{ (-\omega_1 + \omega_2)t + \psi \} - \sin \{ (\omega_2 - \omega_1)t + \psi \}) + \right.$$



$$\begin{aligned}
& 3b_{91} \epsilon_m^2 (\sin \omega_1 t - \sin \{ (2\omega_2 + \omega_1)t + 2\psi \} \\
& - \sin \{ (\omega_1 - 2\omega_2)t - 2\psi \}) + b_{101} \epsilon_m^3 (3/2 \sin (\omega_2 t + \psi) \\
& - \sin(3\omega_2 t + 3\psi)) + \\
& 1/24 [b_{111} \epsilon_m^4 (3/2 - 2 \cos 2\omega_1 t + \cos 4\omega_1 t) + \\
& 4b_{121} \epsilon_m^3 (-3/4 \cos \{ (\omega_1 + \omega_2)t + \psi \} + \\
& 3/4 \cos \{ (\omega_1 - \omega_2)t - \psi \} + \cos \{ (3\omega_1 + \omega_2)t + \psi \} \\
& - \cos \{ (3\omega_1 - \omega_2)t - \psi \}) \\
& + 6b_{131} \epsilon_m^2 (1 + \cos \{ 2(\omega_1 + \omega_2)t + 2\psi \} + \\
& \cos \{ 2(\omega_1 - \omega_2)t - \psi \} - \cos 2\omega_1 t - \cos \{ 2\omega_2 t + 2\psi \}) + \\
& 4b_{141} \epsilon_m^3 (-3/4 \cos \{ (\omega_1 + \omega_2)t + \psi \} + 3/4 \cos \{ (\omega_1 - \omega_2)t - \psi \} + \\
& \cos \{ (3\omega_2 + \omega_1)t + 3\psi \} - \cos \{ (3\omega_2 - \omega_1)t + 3\psi \}) + \\
& b_{151} \epsilon_m^4 (3/2 - 2 \cos(2\omega_2 t + 2\psi) + \cos \{ \omega_2 t + \psi \})] + \dots (1)
\end{aligned}$$

It is seen from the above that the output contains the following difference frequency term:

$$\begin{aligned}
i_{c \text{ diff.}} = & \left[ b_{91} \epsilon_m^2 + 1/16 b_{121} \epsilon_m^3 + 1/16 b_{141} \epsilon_m^3 + \dots \right] \\
& \cos \{ (\omega_1 - \omega_2)t - \psi \} \quad (5)
\end{aligned}$$

In order to simplify further calculations, this equation can be rewritten using rms values.

$$\begin{aligned}
i_{c \text{ diff.}} = & b_{91} \epsilon^2 + 1/16 b_{121} \epsilon^3 + 1/16 b_{141} \epsilon^3 + \dots \quad (6) \\
& \text{freq.}
\end{aligned}$$





### 3. STATE CALCULATIONS

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This coefficient  $\alpha$  is calculated from the plotted results shown in Figure V on p. 417 and Figure VII on p. 420. Equation (5) on p. 42 gives the expression for the difference frequency output current that was measured. From Figure VI, it is seen that for relatively small signals there is a measurable discrepancy from perfect multiplication. Therefore, in this range the error terms can be neglected, and  $\alpha$  is calculated from a direct substitution of values.

$$V_0 = 1.7 \text{ v}$$

$$R_0 = 0.9 \text{ ohms}$$

$$I_0 = 0.0231 \text{ is read.}$$

$$\text{Therefore } 0.0231 = I_0 (1.7) (0.9)$$

$$\text{or } \alpha = .0231$$

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differentiation equation (4) with respect to  $I_0$  gives:

$$\frac{dI_2}{dI_0} = 2I_0 + 2I_1 + \frac{I_2}{I_0} + 2I_2 \frac{1}{I_0} + \epsilon \quad (6)$$

By substituting  $I_0 = 0.0231$  and  $I_1 = 0.001$  it is seen that the variation of  $I_2$  with  $I_0$  decreases for increasing values of  $I_0$ . From equation (7) that  $I_1$  is negligible.

The first part of the paper discusses the general theory of the subject, and the second part discusses the particular case of the subject. The first part is divided into two sections, the first of which discusses the general theory of the subject, and the second of which discusses the particular case of the subject. The second part is divided into two sections, the first of which discusses the general theory of the subject, and the second of which discusses the particular case of the subject.

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From examination of equation (1) on page 42, it is seen that the magnitude of  $s_{12}$  can be calculated from the measured value at the  $(\omega_1 + \omega_2)$  frequency,

$$\text{measured value} = \frac{16}{21} (1) s_{12} \omega_2^2 - \epsilon (1) (1)$$

$$\text{or measured value} = \frac{16}{3} s_{12} \omega_2^2 - \epsilon$$

For  $\omega_c = 2.0$  volts;  $\omega_e = 1.0$  ma; measurement = 0.00054 ma.

$$\text{Therefore } s_{12} = \frac{(0.00054) (3)}{(16) (1.0^2)} = 0.000195$$

For  $\omega_c = 2.5$  volts;  $\omega_e = 1.25$  ma; measurement = 0.00134 ma.

$$\text{Therefore } s_{12} = \frac{(0.00134) (3)}{(16) (1.25^2)} = 0.000206$$

For  $\omega_c = 3.0$  volts;  $\omega_e = 1.50$  ma; measurement = 0.0016 ma.

$$\text{Therefore } s_{12} = \frac{(0.0016) (3)}{(16) (1.5^2)} = 0.000197$$

Therefore  $s_{12}$  has been equal to 0.0002.

### $s_{11}$

Differentiating equation (1) with respect to  $\omega_e$  yields:

$$\frac{d\omega_c}{d\omega_e} = s_{11} \omega_c + s_{12} \omega_c^2 + 3s_{11} \omega_c \omega_e^2 \quad (2)$$

By examining Figure VII, which is a graph of  $\omega_c$  versus  $\omega_e$ , it is seen that the derivative of  $\omega_c$  with respect to  $\omega_e$  decreases as increasing values of  $\omega_e$ . This indicates from equation (2) that  $s_{11}$  is also negative.

Similar to the  $s_{12}$  calculation,  $s_{11}$  can be calculated from the measured value at the  $(\omega_2 - \omega_1)$  frequency.

3. Let  $f(x) = x^2 + 1$  and  $g(x) = x^2 - 1$ . Find  $(f \circ g)(x)$ .

4. Let  $f(x) = x^2 + 1$  and  $g(x) = x^2 - 1$ . Find  $(g \circ f)(x)$ .

5. Let  $f(x) = x^2 + 1$  and  $g(x) = x^2 - 1$ . Find  $(f \circ f)(x)$ .

6. Let  $f(x) = x^2 + 1$  and  $g(x) = x^2 - 1$ . Find  $(g \circ g)(x)$ .

7. Let  $f(x) = x^2 + 1$  and  $g(x) = x^2 - 1$ . Find  $(f \circ g) \circ f(x)$ .

8. Let  $f(x) = x^2 + 1$  and  $g(x) = x^2 - 1$ . Find  $(g \circ f) \circ g(x)$ .

$$(f \circ g)(x) = f(g(x)) = f(x^2 - 1) = (x^2 - 1)^2 + 1 = x^4 - 2x^2 + 1 + 1 = x^4 - 2x^2 + 2$$

9. Let  $f(x) = x^2 + 1$  and  $g(x) = x^2 - 1$ . Find  $(g \circ f) \circ f(x)$ .

$$(g \circ f)(x) = g(f(x)) = g(x^2 + 1) = (x^2 + 1)^2 - 1 = x^4 + 2x^2 + 1 - 1 = x^4 + 2x^2$$

10. Let  $f(x) = x^2 + 1$  and  $g(x) = x^2 - 1$ . Find  $(f \circ g) \circ g(x)$ .

$$(f \circ g)(x) = f(g(x)) = f(x^2 - 1) = (x^2 - 1)^2 + 1 = x^4 - 2x^2 + 1 + 1 = x^4 - 2x^2 + 2$$

11. Let  $f(x) = x^2 + 1$  and  $g(x) = x^2 - 1$ . Find  $(g \circ f) \circ f(x)$ .

12. Let  $f(x) = x^2 + 1$  and  $g(x) = x^2 - 1$ . Find  $(f \circ f) \circ f(x)$ .

$$(f \circ f)(x) = f(f(x)) = f(x^2 + 1) = (x^2 + 1)^2 + 1 = x^4 + 2x^2 + 1 + 1 = x^4 + 2x^2 + 2$$

13. Let  $f(x) = x^2 + 1$  and  $g(x) = x^2 - 1$ . Find  $(g \circ g) \circ g(x)$ .

14. Let  $f(x) = x^2 + 1$  and  $g(x) = x^2 - 1$ . Find  $(f \circ f) \circ g(x)$ .

15. Let  $f(x) = x^2 + 1$  and  $g(x) = x^2 - 1$ . Find  $(g \circ g) \circ f(x)$ .

16. Let  $f(x) = x^2 + 1$  and  $g(x) = x^2 - 1$ . Find  $(f \circ g) \circ f \circ g(x)$ .

17. Let  $f(x) = x^2 + 1$  and  $g(x) = x^2 - 1$ . Find  $(g \circ f) \circ g \circ f(x)$ .

18. Let  $f(x) = x^2 + 1$  and  $g(x) = x^2 - 1$ . Find  $(f \circ f) \circ g \circ f(x)$ .

$$\text{Measured value} = \frac{16}{24} (4) a_{14} e^{-\frac{3}{2}} (1) (1)$$

$$\text{or measured value} = \frac{1}{3} a_{14} e^{-\frac{3}{2}}$$

$$\text{for } E_c = 2.0 \text{ volts; } t_e = 1.04 \text{ ms; measurement} = 0.00061 \text{ ma.}$$

$$\text{Therefore } a_{14} = \frac{(0.00061) (3)}{(2) (1.13)} = 0.00081$$

$$\text{for } E_c = 2.5 \text{ volts; } t_e = 1.25 \text{ ms; measurement} = 0.0013 \text{ ma.}$$

$$\text{Therefore } a_{14} = \frac{(0.0013) (3)}{(2.5) (1.75)} = 0.0009$$

$$\text{for } E_c = 2.5 \text{ volts; } t_e = 1.562 \text{ ms; measurement} = 0.0025 \text{ ma.}$$

$$\text{Therefore } a_{14} = \frac{(0.0025) (3)}{(2.5) (2.2)} = 0.000727$$

Therefore  $a_{14}$  was determined to be 0.00081.

$$f(x) = \frac{1}{2} \left( \frac{1}{x} + \frac{1}{x^2} \right) \quad f'(x) = -\frac{1}{2} \left( \frac{1}{x^2} + \frac{2}{x^3} \right)$$

$$f'(x) = -\frac{1}{2} \left( \frac{1}{x^2} + \frac{2}{x^3} \right) = -\frac{1}{2} \left( \frac{x+2}{x^3} \right)$$

$$f'(x) = -\frac{1}{2} \left( \frac{x+2}{x^3} \right) = -\frac{1}{2} \left( \frac{1+2}{1^3} \right) = -\frac{3}{2}$$

$$f'(1) = -\frac{3}{2} = -1.5$$

$$f'(1) = -1.5 \quad f(1) = \frac{1}{2} \left( \frac{1}{1} + \frac{1}{1^2} \right) = 1$$

$$f(1) = 1 \quad f'(1) = -1.5$$

$$f(1) = 1 \quad f'(1) = -1.5$$

$$f(1) = 1 \quad f'(1) = -1.5$$

$$f(1) = 1 \quad f'(1) = -1.5$$

C. ORIGINAL DATAWestern Electric, Type 1B93. Circuit Diagram, Figure 17 $e_o = -4$  volts,  $I_{E0} = 1.5$  ma,  $I_{C0} = 5.37$  ma.April 16, 1964.

$f_c = 100$ cps across 1k	$f_c = 300$ cps	$f_c = 100$ cps across 750 $\Omega$	$f_c = 1500$ cps across 750 $\Omega$	$f_c = 100$ cps across 750 $\Omega$
50 mv	50 mv	6.0 mv	Not	Not
	750	8.6	measured	measured
	1000	10.7	"	"
	1250	12.7	"	"
	1500	13.5	1.7 mv	"
	2000	13.	2.4	"
100	500	11.4	Not	Not
	750	17.5	measured	measured
	1000	21.0	"	"
	1250	25.0	"	"
	1500	27.0	2.7	3.13 mv
	2000	27.5	5.0	5.17
150	500	17.4	Not	Not
	750	21.5	measured	measured
	1000	31	"	"
	1250	35	"	"
	1500	40	15.3	17.1
	2000	40	15.	17.5
200	500	22	Not	Not
	750	22	measured	measured
	1000	30	"	"
	1250	37	"	"
	1500	41	1.7	1.7
	2000	41	1.7	1.7





General Electric padded, modified transistor.

Circuit Diagram, Figure 111

$V_{CC} = -4$  volts,  $I_{EO} = 1.5$  ma.

April 4, 1952

$f_{c1} = 100$  cps across  $9.0\Omega$      $f_{c2} = 350$  cps across  $100\Omega$      $f_{c3} = 1000$  cps across  $100\Omega$      $f_{c4} = 1500$  cps across  $100\Omega$      $f_{c5} = 10000$  cps across  $100\Omega$

1 volt	0.5 volts	0.15 mv	too small to read	too small to read
1	1.0	0.27	"	"
1	1.25	0.30	"	"
1	2.0	0.53	"	"
1	2.5	0.57	"	"
2	0.5	0.30	"	"
2	1.0	0.395	Not measured	Not measured
2	1.5	0.52	"	"
2	2.0	1.20	"	"
2	2.5	1.43	"	"
2	2.0	1.57	"	"
4	0.5	0.31	"	"
4	1.0	1.21	"	"
4	1.5	1.3	"	"
4	2.0	1.41	"	"
4	2.5	2.74	"	"
4	2.0	3.20	"	"
6	0.5	0.34	"	"
6	1.0	1.2	"	"
6	1.5	2.0	"	"
6	2.0	2.35	"	"
6	2.5	1.70	"	"
6	2.0	1.70	"	"
8.5	0.5	1.31	"	"
8.5	1.0	1.52	"	"
8.5	1.5	2.70	"	"
8.5	2.0	1.00	"	"
8.5	2.5	1.70	"	"
10.0	2.0	2.70	0.15 mv	0.15 mv
12.0	2.0	6.50	0.15	0.15
15.0	2.0	1.00	0.15	0.15

1. The first part of the report is devoted to a general description of the project.

2. The second part of the report is devoted to a detailed description of the project.

3. The third part of the report is devoted to a detailed description of the project.

4. The fourth part of the report is devoted to a detailed description of the project.

5. The fifth part of the report is devoted to a detailed description of the project.

Item	Quantity	Unit	Value	Total
1. Material	100	kg	10.00	10.00
2. Labor	100	hr	10.00	10.00
3. Equipment	100	hr	10.00	10.00
4. Material	100	kg	10.00	10.00
5. Labor	100	hr	10.00	10.00
6. Equipment	100	hr	10.00	10.00
7. Material	100	kg	10.00	10.00
8. Labor	100	hr	10.00	10.00
9. Equipment	100	hr	10.00	10.00
10. Material	100	kg	10.00	10.00
11. Labor	100	hr	10.00	10.00
12. Equipment	100	hr	10.00	10.00
13. Material	100	kg	10.00	10.00
14. Labor	100	hr	10.00	10.00
15. Equipment	100	hr	10.00	10.00
16. Material	100	kg	10.00	10.00
17. Labor	100	hr	10.00	10.00
18. Equipment	100	hr	10.00	10.00
19. Material	100	kg	10.00	10.00
20. Labor	100	hr	10.00	10.00
21. Equipment	100	hr	10.00	10.00
22. Material	100	kg	10.00	10.00
23. Labor	100	hr	10.00	10.00
24. Equipment	100	hr	10.00	10.00
25. Material	100	kg	10.00	10.00
26. Labor	100	hr	10.00	10.00
27. Equipment	100	hr	10.00	10.00
28. Material	100	kg	10.00	10.00
29. Labor	100	hr	10.00	10.00
30. Equipment	100	hr	10.00	10.00
31. Material	100	kg	10.00	10.00
32. Labor	100	hr	10.00	10.00
33. Equipment	100	hr	10.00	10.00
34. Material	100	kg	10.00	10.00
35. Labor	100	hr	10.00	10.00
36. Equipment	100	hr	10.00	10.00
37. Material	100	kg	10.00	10.00
38. Labor	100	hr	10.00	10.00
39. Equipment	100	hr	10.00	10.00
40. Material	100	kg	10.00	10.00
41. Labor	100	hr	10.00	10.00
42. Equipment	100	hr	10.00	10.00
43. Material	100	kg	10.00	10.00
44. Labor	100	hr	10.00	10.00
45. Equipment	100	hr	10.00	10.00
46. Material	100	kg	10.00	10.00
47. Labor	100	hr	10.00	10.00
48. Equipment	100	hr	10.00	10.00
49. Material	100	kg	10.00	10.00
50. Labor	100	hr	10.00	10.00
51. Equipment	100	hr	10.00	10.00
52. Material	100	kg	10.00	10.00
53. Labor	100	hr	10.00	10.00
54. Equipment	100	hr	10.00	10.00
55. Material	100	kg	10.00	10.00
56. Labor	100	hr	10.00	10.00
57. Equipment	100	hr	10.00	10.00
58. Material	100	kg	10.00	10.00
59. Labor	100	hr	10.00	10.00
60. Equipment	100	hr	10.00	10.00
61. Material	100	kg	10.00	10.00
62. Labor	100	hr	10.00	10.00
63. Equipment	100	hr	10.00	10.00
64. Material	100	kg	10.00	10.00
65. Labor	100	hr	10.00	10.00
66. Equipment	100	hr	10.00	10.00
67. Material	100	kg	10.00	10.00
68. Labor	100	hr	10.00	10.00
69. Equipment	100	hr	10.00	10.00
70. Material	100	kg	10.00	10.00
71. Labor	100	hr	10.00	10.00
72. Equipment	100	hr	10.00	10.00
73. Material	100	kg	10.00	10.00
74. Labor	100	hr	10.00	10.00
75. Equipment	100	hr	10.00	10.00
76. Material	100	kg	10.00	10.00
77. Labor	100	hr	10.00	10.00
78. Equipment	100	hr	10.00	10.00
79. Material	100	kg	10.00	10.00
80. Labor	100	hr	10.00	10.00
81. Equipment	100	hr	10.00	10.00
82. Material	100	kg	10.00	10.00
83. Labor	100	hr	10.00	10.00
84. Equipment	100	hr	10.00	10.00
85. Material	100	kg	10.00	10.00
86. Labor	100	hr	10.00	10.00
87. Equipment	100	hr	10.00	10.00
88. Material	100	kg	10.00	10.00
89. Labor	100	hr	10.00	10.00
90. Equipment	100	hr	10.00	10.00
91. Material	100	kg	10.00	10.00
92. Labor	100	hr	10.00	10.00
93. Equipment	100	hr	10.00	10.00
94. Material	100	kg	10.00	10.00
95. Labor	100	hr	10.00	10.00
96. Equipment	100	hr	10.00	10.00
97. Material	100	kg	10.00	10.00
98. Labor	100	hr	10.00	10.00
99. Equipment	100	hr	10.00	10.00
100. Material	100	kg	10.00	10.00

General Electric padder, modified transistor.

Circuit diagram, micro film

$V_{cc} = -4$  volts,  $I_{EP} = 1.5$  ma.

April 25, 1962.

$\omega_1$	$\omega_2$	$V_{I_E}$ across $1.0$	$V_C$	$V_{I_C}$ diff. freq. across $1.0\Omega$
350 cps	450 cps	2 volts	0.5 volts	.305 mv
			1.0	.305
			1.5	.310
			2.0	1.175
			2.5	1.12
			3.0	.610
			1.0	1.20
			1.5	1.10
			2.0	2.35
			2.5	2.73
3.7 kc	4.7 kc		.5	.275
			1.0	.325
			1.5	.375
			2.0	1.10
			2.5	1.21
			3.0	1.30
			1.0	1.10
			1.5	1.30
			2.0	1.35
			2.5	2.1
5.75 kc	7.25 kc		1.0	.35
			1.5	.375
9.75 kc	12.45 kc		1.0	.35
			1.5	.375
15 kc	15 kc		1.0	.125
			1.5	.375
			1.0	.375
			1.5	.375
			2.0	.375
			2.5	.375
			3.0	.375
			1.0	.375
			1.5	.375
			2.0	.375
			2.5	.375
			3.0	.375

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1. The first part of the report is a summary of the work done during the year. It is a brief statement of the results of the work, and is intended to give a general idea of the progress made.

2. The second part of the report is a detailed account of the work done during the year. It is a full and complete statement of the results of the work, and is intended to give a detailed account of the progress made.

3. The third part of the report is a summary of the work done during the year. It is a brief statement of the results of the work, and is intended to give a general idea of the progress made.

4. The fourth part of the report is a summary of the work done during the year. It is a brief statement of the results of the work, and is intended to give a general idea of the progress made.

5. The fifth part of the report is a summary of the work done during the year. It is a brief statement of the results of the work, and is intended to give a general idea of the progress made.

6. The sixth part of the report is a summary of the work done during the year. It is a brief statement of the results of the work, and is intended to give a general idea of the progress made.

7. The seventh part of the report is a summary of the work done during the year. It is a brief statement of the results of the work, and is intended to give a general idea of the progress made.

8. The eighth part of the report is a summary of the work done during the year. It is a brief statement of the results of the work, and is intended to give a general idea of the progress made.

9. The ninth part of the report is a summary of the work done during the year. It is a brief statement of the results of the work, and is intended to give a general idea of the progress made.

10. The tenth part of the report is a summary of the work done during the year. It is a brief statement of the results of the work, and is intended to give a general idea of the progress made.











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